Casual Microstrip Design

WD5IWN

2023-03-05

Contents

Some years ago (~30), I was dabbling in the mysterious world of microwave design (and I intend to again, now that I have more time), and was looking at various formulas to determine dimensions for microstrip widths and lengths. I had two paths to investigate, namely, references by (Edwards and Steer 2016) and (Pozar 2012). That's because those were the two reference books I had (now with more recent copies). Terry Edward's book seems more in-depth with a nuts-and-bolts approach, where David Pozar's volume is more a macro-level dissertation of the subject. Both books are chock full of useful information for microwave design, and generally follow the same paths, and although the results are similar, there are slight differences in final dimensions.

1: Edwards: Foundations for Microstrip Circuit Design.

Firstly, I will depict the formulas I have used for years for determining dimensions. I generally neglect dispersion, fringing effects, etc., especially at lower frequencies. Chapter 7 of (Edwards and Steer 2016) covers dispersion at higher frequencies. For in-depth analysis, there is no replacement for actually studying the book.

The formulas I show here, from Edward's book, are closed formulas for static-TEM[1] designs. There are two types of microstrips, high impedance and low impedance, and the formulas are different. Narrow strips

[1] Transverse Electromagnetic

are defined where desired impedance, $Z_o > (44-2e_r)\Omega$. Wide strips are where impedance, $Z_o < (44-2e_r)\Omega$.

High impedance, narrow strips. For determining the ratio of microstrip width (w) to substrate thickness (h), we use the formulas below. As H' is used in the w/h formula, we determine it first:

$$H' = \frac{Z_o\sqrt{2(e_r+1)}}{119.9} + \frac{1}{2}\left(\frac{e_r-1}{e_r+1}\right)\left(ln\frac{\pi}{2} + \frac{1}{e_r}ln\frac{4}{\pi}\right) \tag{1}$$

and then:

$$\frac{w}{h} = \left(\frac{e^{H'}}{8} - \frac{1}{4e^{H'}}\right)^{-1} \tag{2}$$

where Euler's number, e = 2.718281828, and e_r is relative permittivity[2] of the board substrate. Now, to determine the effective relative permittivity:

$$E_{eff} = \frac{e_r+1}{2}\left[1 - \frac{1}{2H'}\left(\frac{e_r-1}{e_r+1}\right)\left(ln\frac{\pi}{2} + \frac{1}{e_r}ln\frac{4}{\pi}\right)\right]^{-2} \tag{3}$$

Low impedance, wide strips. For wide strips, the formula is a bit different. As above we first determine one of the variables d_e. The w/h formula from Edwards has two variables, d_{er} and d_{e1}. However, the difference in output is barely noticeable, so I will make a little jump here and use $d_e = \frac{59.95\pi^2}{Z_o\sqrt{e_r}}$ for both in the below w/h formula. I decided this by cross-referencing the (Wedge, Compton, and Rutledge 1991) book for CAD microwave design, the ubiquitous PUFF.

$$\frac{w}{h} = \frac{2}{\pi}[(d_e-1) - ln(2d_e-1)] + \frac{(e_r-1)}{\pi e_r}\left[ln(d_e-1) + 0.293 - \frac{0.517}{e_r}\right] \tag{4}$$

Final dimensions. Determining the microstrip width is simply substrate thickness (h) divided by inverse w/h ratio, $w = \frac{h}{w/h^{-1}}$. As I wanted to make it easier to program this on my handheld calculator, I broke up the following formula as so:

$$\begin{aligned} A = 1.0 + &\left[ln\left(e^{4ln\frac{w}{h}} + \left(\frac{w}{h}/52\right)^2\right) / \left(e^{4ln\left(\frac{w}{h}\right)} + 0.432/49\right)\right] \\ &+ \left[ln\left((1 + e^{3ln\frac{w/h}{18.1}}\right)/18.7\right] \end{aligned} \tag{5}$$

$$B = 0.564e^{0.053ln\frac{e_r-0.9}{e_r+3.0}} \tag{6}$$

$$C = \frac{e_r+1.0}{2} + \left[\frac{e_r-1.0}{2} \times e^{-ABln\left(1.0+\frac{10}{w/h}\right)}\right] \tag{7}$$

Finally, the microstrip length (ℓ) is deduced by the desired phase angle (ϕ) such:

$$\ell = (\phi \times \frac{c}{freq/\sqrt{C}}/360) \times 1000 \tag{8}$$

where lightspeed (c) = 2.99792458e[8].

So now, let's use an example to show the process and determine some useful results. We could use an alumina substrate with thickness (h) 0.6mm, relative permittivity (e_r) 9.8 and dielectric loss tan (δ) 0.001, at a frequency of 2 GHz with a phase angle (ϕ) of 90° and input impedance (Z_o) of 50 Ω.

[2] Relative Permittivity is defined as the ratio of the actual or absolute permittivity of a medium to the absolute permittivity of vacuum.

Those parameters give us the following results:

- w/h = 0.976
- E_{eff} = 6.555
- width = 0.586 mm
- length = 14.636 mm

At higher frequencies, dispersion calculations would be required, detailed in Chapter 7 of (Edwards and Steer 2016).

2: Pozar: Microwave Engineering

So, from Pozar's formulas, where the w/h ratio is less than 2, we use the following:

$$\frac{w}{h} = \frac{8e^A}{e^{2A} - 2} \tag{9}$$

where

$$A = \frac{Z_o}{60}\sqrt{\frac{e_r + 1}{2}} + \frac{e_r - 1}{e_r + 1}\left(0.23 + \frac{0.11}{e_r}\right)$$

If the w/h ratio is greater than 2, we use this formula:

$$\frac{w}{h} = \frac{2}{\pi}\left[B - [@puff1991]1 - ln(2B - 1) + \frac{e_r - 1}{2e_r}\left(ln(B - 1) + 0.39 - \frac{0.61}{e_r}\right)\right] \tag{10}$$

where

$$B = \frac{377\pi}{2Z_o\sqrt{e_r}}$$

Final dimensions. We once again determine the width as such:

$$w = \frac{h}{w/h^{-1}}\ or\ w = \frac{w}{h} \times h \tag{11}$$

And, for a slightly different way to determine length, we first find the effective dielectric constant (e_e) of the microstrip line with:

$$e_e = \frac{e_r + 1}{2} + \frac{e_r - 1}{2} \times \frac{1}{\sqrt{1 + 12h/w}} \tag{12}$$

and with $k_o = \frac{2\pi f}{c}$, we find the length:

$$\ell = \frac{\phi(\pi/180^o)}{\sqrt{e_e}k_o} \tag{13}$$

For a higher frequency example, we again use an alumina substrate with thickness (h) 0.6mm, relative permittivity (e_r) 9.8 and dielectric loss tan (δ) 0.001, at a frequency of 10 GHz with a phase angle (ϕ) of 270° and input impedance (Z_o) of 50 Ω. This gives the following results:

- w/h = 0.975
- E_{eff} = 6.508
- width = 0.4876 mm
- length = 8.8136 mm

3: Exploring bandpass filters.

Now we enter into the whole purpose for this article, determining the impedance values for a bandpass filter. We will use an example found elsewhere so we have a cross-reference for the results. This is a design for a GPS bandpass filter[3]. The values used are for a third-order Chebyshev filter design for a frequency of 1575 MHz using a Rogers 4003 board, whereas we are using FR4 board. So, the end result is somewhat different.

- Board:
  - $e_r = 4.5$.
  - substrate = 1.57.
  - copper = 35 um.
- Center frequency = 1575 MHz.
- Lower frequency = 1550 MHz.
- Upper frequency = 1600 MHz.
- Ripple factor = 0.01.
- Filter order = 3.
- System impedance (Z_O) = 50 ohms.

At this higher frequency, losses and ripple are difficult to control for FR4 board. A better board would be Rogers 4003. At any rate, the first factors we need to determine are the normalized values for the components. These are the formulae:

$$beta = log\left(\frac{\cosh(ripple/17.37)}{\sinh(ripple/17.37}\right) \tag{14}$$

$$y = \sinh\left(\frac{beta}{2 * order}\right) \tag{15}$$

There must be one more pair than the desired filter order (i.e., N of 3 = 4 pairs). Two additional calculations are required for each pair. An easy way to store those values in R is a matrix or data.frame.

The additional formulae using count=1 to N are:

$$x_{count} = \sin\left(\frac{2 * count * \pi}{2N}\right) \tag{16}$$

$$z_{count} = y^2 + \sin\left(\frac{count * \pi}{N}\right)^2 \tag{17}$$

After looping through each count and saving the values, we then determine the first normalized value as so:

$$G_1 = 2 * x_1/y$$

For the remaining G values:

$$G_{count} = \frac{4 * x_{count-1} * x_{count}}{z_{count-1} * G_{count-1}} \tag{18}$$

In the above referenced article, another program (faisyn) was used to determine the normalized values, which broke the calculation chain. That's why I have recreated the formulae here. The R_{load} value for odd-N orders will be 1. The normalized values are:

- $g_1 = 0.6291$
- $g_2 = 0.9702$
- $g_3 = 0.6291$
- R_{load} 1

[3] Part of an article by Gunthard Kraus, DG8GB.

Next step was to determine the fractional bandwidth simply as:

$$\triangle = \frac{f_{max} - f_{min}}{f_{mean}} = 0.031746 \tag{19}$$

This value is used in the admittance inverter constant calculations for each pair.

$$Z_o.J_1 = \sqrt{\frac{\pi * \triangle}{2 * g_1}} = 0.28154 \tag{20}$$

$$Z_o.J_2 = \frac{\pi * \triangle}{2 * \sqrt{g_1 * g_2}} = 0.06383 \tag{21}$$

$$Z_o.J_3 = \frac{\pi * \triangle}{2 * \sqrt{g_2 * g_3}} = 0.06383 \tag{22}$$

And finally:

$$Z_o.J_4 = \sqrt{\frac{\pi * \triangle}{2 * g_3 * g_4}} = 0.28154 \tag{23}$$

Each line pair even/odd impedances are calculated as such:

$$Z_{EVEN} = Z_O * |1 + Z_o.J_N + (Z_o.J_N)^2| \tag{24}$$

$$Z_{ODD} = Z_O * |1 - Z_o.J_N + (Z_o.J_N)^2| \tag{25}$$

That gives us the even/odd impedances in ohms (@ 90°) for our microstrip line pairs.

- Even Odd
- 68.04 39.89
- 53.40 47.01
- 53.40 47.01
- 68.04 39.89

We now have the impedance values, even and odd, we need to create an actual filter design. We can use (Wedge, Compton, and Rutledge 1991) to check responses and to adjust the values for filter shape, etc.

PUFF CAD program. If we enter these values into PUFF as ‘clines,’ we can determine the performance of the filter. We set up PUFF [BOARD] factors from the list above, then create two ‘cline’ entries in the [PARTS] box, formatted as: *cl*! 68.04Ω 39.89Ω 90^o and *cl*! 53.40Ω 47.01Ω 90^o. The area for entries in the [PARTS] is tight, so the above spaces are not necessary, but are shown for clarity. In PUFF, the symbols for ohms (Ω) and degrees (°) are entered with CTRL-O and CTRL-D. The ‘!’ in the ‘cline’ allows PUFF to compensate for dispersion losses and other factors, and will modify the actual values. So it is necessary to adjust the values by pressing ‘=’ while in the [PARTS] box on the desired part. The actual values are shown directly above in a red box. Change the ‘cline’ values to bring those values back to the calculated values.

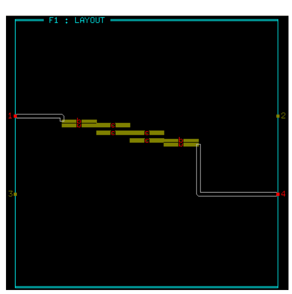


Figure 1: Layout diagram.

Figure 1 above shows the 'cline' layout, where the first 'cline' is at each end, and the second 'cline' is in the center twice. Connection to port 4 is only to make the connections look clearer. Normal values would be S11 and S21, but S41 looks neater, and doesn't affect the actual parameters. The return reflection shows -1.31 dB loss at 0.1° phase angle. These values are after adjustment. To see the physical dimensions, remove the '!' from the 'cline' to read the width and length, again by pressing '=.'

For a real circuit, fringing effect end corrections must be applied, which will shorten the lines a bit. For coupled lines, the value would be negative, but only **half** of the calculated length. Determine the length extension (ℓ) using:

$$\frac{l}{h} = 0.412\left(\frac{e_{re}+0.3}{e_{re}-0.258}\right)\left(\frac{w/h+0.262}{w/h+0.813}\right) \tag{26}$$

where 'h' is substrate thickness. The value, 'e_{re}, referenced in (Hammerstad and Bekkadal 1975) (which is out of print), is vague, but there is a chart (See Figure 2 below) in Chapter 7 of the PUFF manual which allows determining length correction ratio, given Z_o and e_r values, and is much easier than the above formula. Then the extension, $\ell_{eo} = \frac{ratio \times dielectric}{2}$ for the half value.

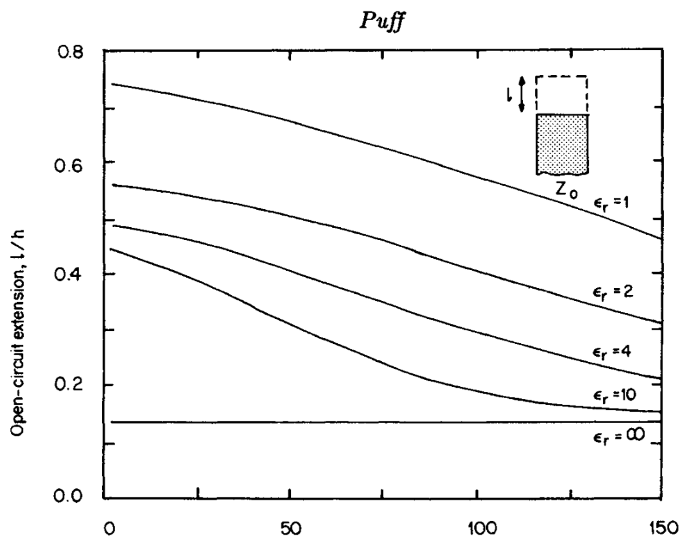


Figure 2: Table 7.2.

Further study shows the above empirical formula is also depicted in Chapter 9 of (Edwards and Steer 2016), but where 'e_{re} is replaced with E_{eff}. It also indicates errors can exceed 5%, which can be significant for filter design.

$$\ell_{eo} = 0.412h\left(\frac{E_{eff}+0.3}{E_{eff}-0.258}\right)\left(\frac{w/h+0.262}{w/h+0.813}\right) \tag{27}$$

where E_{eff} is defined in Section 1, Eq. (3) above. So, using a board of e_r=9.8, h=0.813 mm, a 90° line at 1575 MHz and Z_o=50 ohms gives a ratio of 0.310 which coincides with the above chart. This gives an extension of 0.126 to be subtracted from the open end of a microstrip.

It is worthwhile to acquire a copy of the PUFF manual, if only for the information. PUFF is released in the public domain under the GPL license, and can be acquired for both PC and Linux operating systems. The manual copy usually comes with the package[4].

[4] The PUFF package for Linux is at https://www.pa3fwm.nl/software/puff/

4. Designer's Guide

Here I will expand a bit on an article I saw entitled, "A Designer's Guide to Microstrip Design," and attempt to recreate a portion of the article[5], (Bahl and Trivedi 1977), from the May 1977 issue of Microwaves Journal. I originally came across the reference in (Pozar 2012) Chapter 3, where I was investigating frequency effects on microstrip lines, and wanted to repeat some of the explanation and formulae here.

One area I found interesting was the formulae for Z_o and E_{eff} and the variance as the impedance changes. The width of a microstrip is primarily determined by the substrate thickness and system impedance, whereas the length is a function of frequency. Obviously, frequency plays into all the factors, and has the most impact as it reaches the 2 GHz and above range.

Firstly, a bit of background. Since field lines between the strip and the ground plane are not contained entirely in the substrate, the propagating mode along the strip is not purely transverse electromagnetic (TEM) but quasi-TEM. Assuming the quasi-TEM mode of propagation, the phase velocity in microstrip is given by

$$V_p = \frac{c}{\sqrt{\varepsilon_{eff}}} \tag{28}$$

where c is the velocity of light[6], and ε_{eff} is the effective dielectric constant of the substrate material. The effective dielectric constant is lower than the relative dielectric constant, ε_r, of the substrate, and takes into account external fields. The wavelength, λ_g, in microstrip line is given by

$$\lambda_g = \frac{V_p}{f}\ (in\ m/s) \tag{29}$$

where f is frequency. The characteristic impedance of the transmission line is given by

$$Z_o = \frac{1}{V_p C} \tag{30}$$

where C is the capacitance per unit length of the line. The analysis for the evaluation of ε_{eff} and C based on quasi-TEM mode is fairly accurate at lower microwave frequencies. At higher frequencies, the ratio of longitudinal-to-traverse electric field components becomes significant and the propagating mode can no longer be considered quasi-TEM. Analysis of this "hybrid mode" is far more rigorous.

Closed-form expressions. Closed form expressions by Hammerstad[7] for Z_o and ε_{eff} include useful relationships defining both characteristic impedance and effective dielectric constant:
For W/h $\leq$ 1,

$$Z_o = \frac{60}{\sqrt{\varepsilon_{eff}}} ln(8h/W + 0.25W/h) \tag{31}$$

where:

$$\varepsilon_{eff} = \frac{e_r + 1}{2} + \frac{e_r - 1}{2}[(1 + 12h/W)^{-1/2} + 0.04(1 - W/h)^2] \tag{32}$$

For W/h $\geq$ 1,

$$Z_o = \frac{120\pi/\sqrt{\varepsilon_{eff}}}{W/h + 1.393 + 0.667 ln(W/h + 1.444)} \tag{33}$$

where:

$$\varepsilon_{eff} = \frac{e_r + 1}{2} + \frac{e_r - 1}{2}(1 + 12h/W)^{-1/2} \tag{34}$$

[5] Submitted by Dr. I. J. Bahl, and D. K. Trivedi, Research Engineers, Indian Institute of Technology, Advanced Centre For Electronic Systems, department of Electrical Engineering, Kanpur-208016, India.

[6] Lightspeed is 2.99792458e8 m/s.

[7] E.O Hammerstad, "Equations For Microstrip Circuit Design," Proc. European Microwave Conference, Hamburg (Germany), pp. 268-272, (September 1975).

Hammerstad notes that the maximum relative error in ε_{eff} and Z_o is less than ± 0.5 percent and 0.8 percent, respectively, for $0.05 \leq W/h \leq 20$ and $\varepsilon_r \leq 16$. His expressions for W/h in terms of Z_o and ε_r are: For $W/h \leq 2$,

$$W/h = \frac{8e^A}{e^{2A} - 2} \tag{35}$$

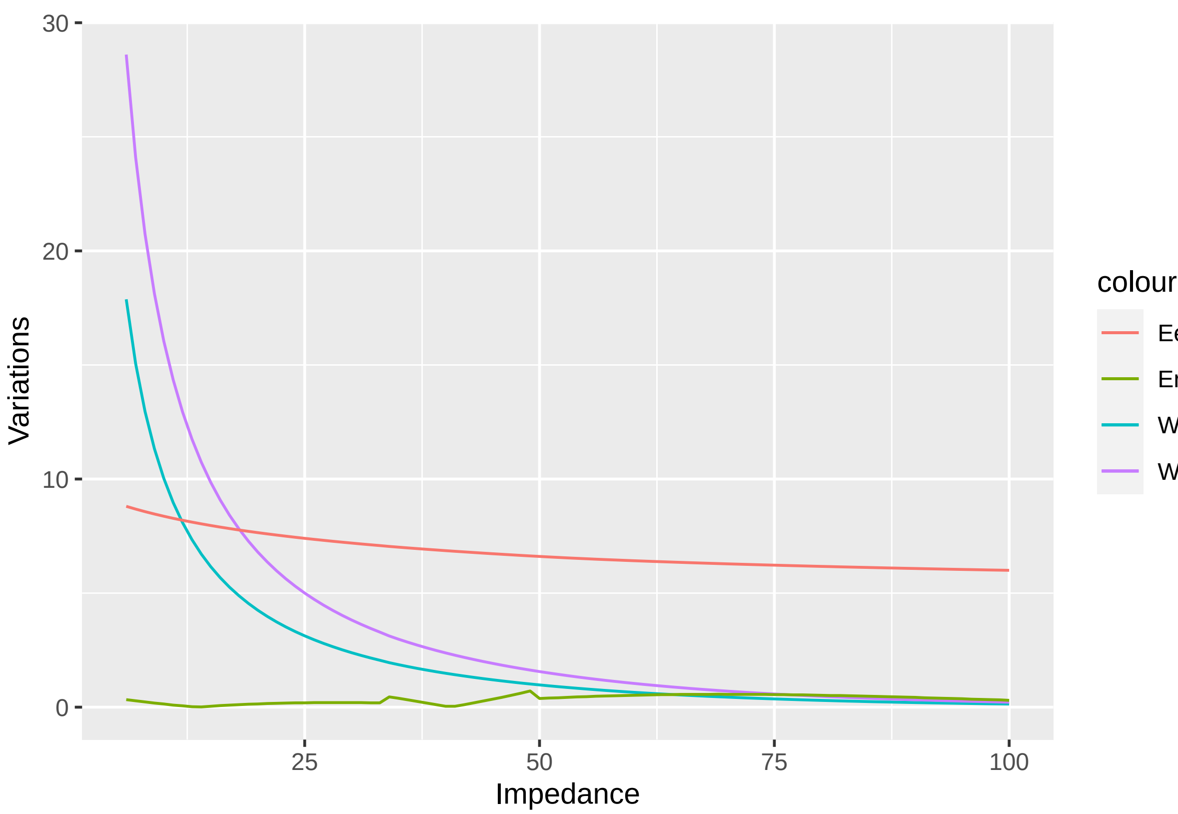
For $W/h \geq 2$,

$$W/h = \frac{2}{\pi}[B - 1 - ln(2B - 1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left(ln(B - 1) + 0.39 - \frac{0.61}{\varepsilon_r}\right)] \tag{36}$$

where:

$$A = \frac{Z_o}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}(0.23 + 0.11/\varepsilon_r)$$
$$B = \frac{377\pi}{2Z_o\sqrt{\varepsilon_r}}$$

These expressions provide the same accuracy as the previous four expressions. So, if we provide a situation where we vary input impedance, Z_o, and using a substrate $e_r = 9.8$ with a thickness of 1.6 mm, we see how the E_{eff} and W/h ratio varies.



The results discussed above assume a two-dimensional strip conductor. But in practice, the strip is three-dimensional; its thickness must be considered. However, when $t/h \leq 0.005$, $2 \leq \varepsilon_r \leq 10$, and $0.1 \leq W/h \leq 5$, the agreement between experimental and theoretical (t/h=0) results is excellent.

Conductor thickness. I wanted to explore the differences when using the modified formulas that include the third dimension of a microstrip, the thickness of the conductor.

Figure 3: Microstrip.

The zero-thickness (t/h=0) formulas given above can also be modified to consider the thickness of the strip when the strip width, W, is replaced by an effective strip width, W_e. Expressions for W_e are:
For $W/h \geq 1/2\pi$,

$$\frac{W_e}{h} = \frac{W}{h} + \frac{t}{\pi h}\left(1 + ln\frac{2h}{t}\right) \tag{37}$$

For $W/h \leq 1/2\pi$,

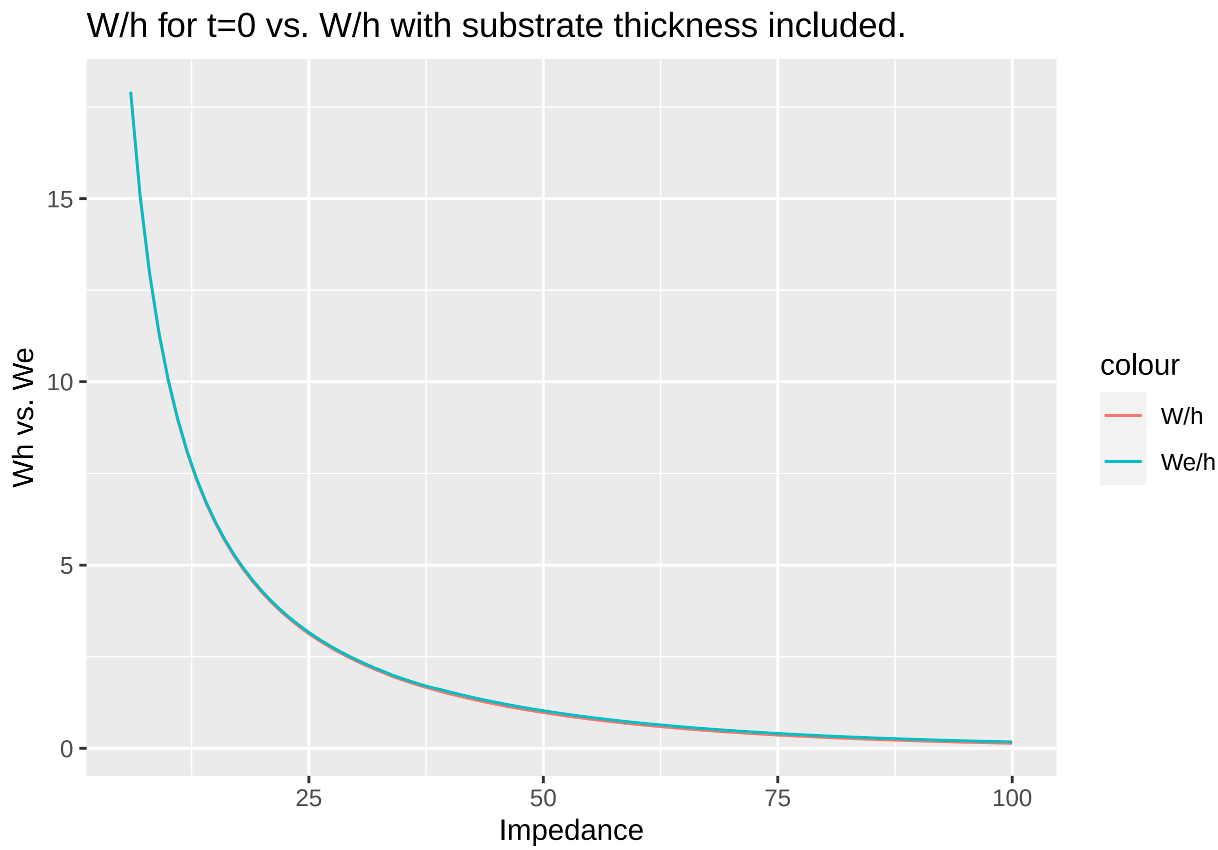
$$\frac{W_e}{h} = \frac{W}{h} + \frac{t}{\pi h}\left(1 + ln\frac{4\pi W}{t}\right) \tag{38}$$

Additional restrictions for applying these equations are $t \leq h$ and $t < W/2$. Typical strip thickness varies from 0.0002 to 0.0005 inch (5.1 um to 12.7 um) for metallized alumina substrate, and from 0.001 to 0.003 inch (25 um to 76 um)[8] for low-dielectric substrates.

Microwave enclosures tend to lower impedance and effective dielectric constant. But when the ratio of the distance between the lower and upper walls to substrate thickness is larger than five, and the sidewall spacing is five times the strip width, the enclosure effect is negligible on microstrip characteristics.

For an example, let's use the chart above, but show the difference when using the conductor thickness. We will use the same substrate values: $\varepsilon_r = 9.8, h = 1.6$, but adding the conductor thickness, t=35 um and an impedance range of $50 \leq Z_o \leq 100\Omega$ for better definition.

[8] 35 um is average thickness for 1 oz copper traces.



Using these formulas, the above chart shows there is negligible difference between W/h and W_e/h. The overall difference, using the mean values, is shown in the chart title. However, this could be significant, especially if impedance matching is critical, or at higher frequencies.

Dispersion. I wish to expound (if I can) on variances of E_{eff} at frequency, but also adding some explorations of frequency effects, specifically where dispersion effects come into play and the cutoff frequency below where they may be neglected.

First, the formula to determine the cutoff frequency for certain substrate values and impedance, Z_o. Frequency changes seem to have small effect on E_{eff} and Z_o. However, to determine the cutoff frequency where they may be neglected, examine,

$$f_o(GHz) = 0.3\sqrt{\frac{Z_o}{h\sqrt{\varepsilon_r - 1}}}\,(h\ in\ cm) \tag{39}$$

The analytical expression for dispersion by Getsinger[9] is given by,

$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}}{1 + G(f/f_p)^2} \tag{40}$$

where:

$$f_p = \frac{Z_o}{8\pi h} \qquad G = 0.6 + 0.009Z_o$$

Here, frequency, f, is in GHz and substrate thickness, h, in cm. So let's see if we can develop a chart showing the variances of line width versus different permittivities for substrate thicknesses, h, of 1.6 and 0.813 mm. Standard FR4 boards and some Rogers boards use this common thickness. At higher frequencies, thinner substrates work better. I will attempt to display the results in chart format.

[9] W.J. Getsinger, "Microstrip Dispersion," Proc. IEEE, Vol. 60, pp. 144-146, (January, 1972).

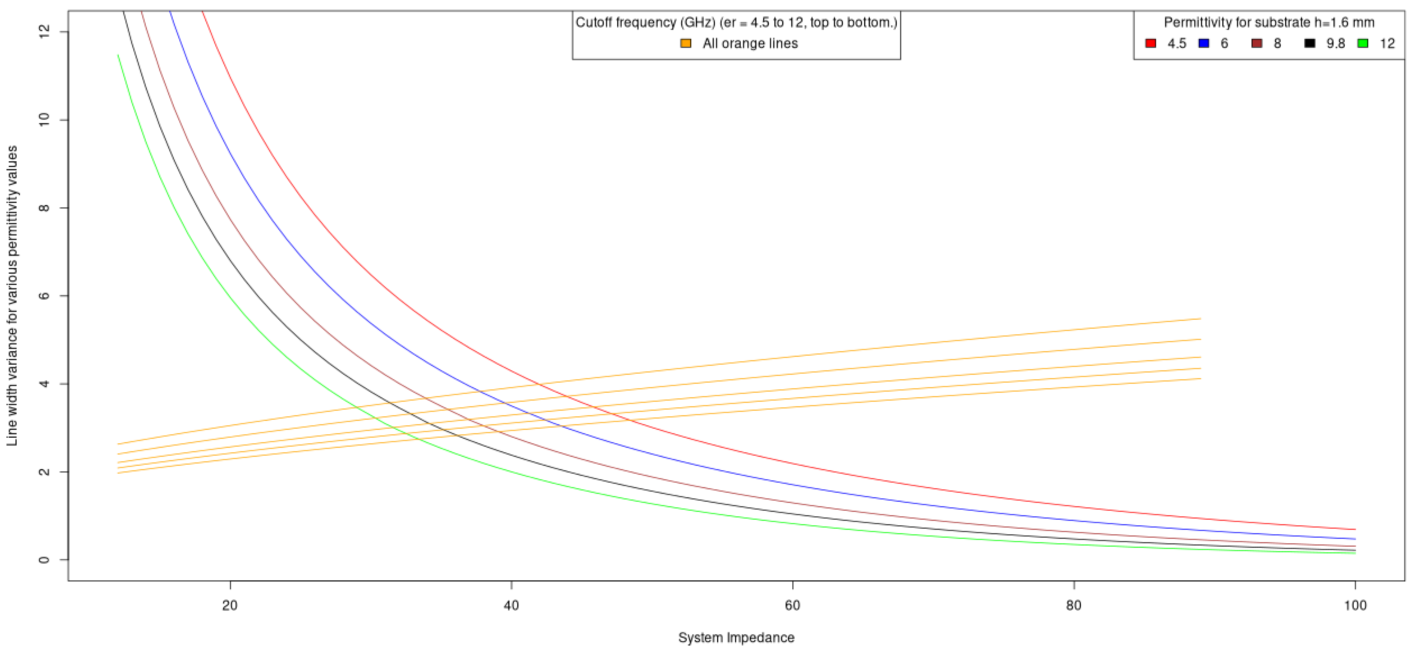


Figure 4: Line width variances for h=1.6.

This first chart shows the changes for substrate = 1.6 mm. Next is showing widths on a 0.813 mm board. Easily seen is the thinner board allows thinner lines for the same impedance values.

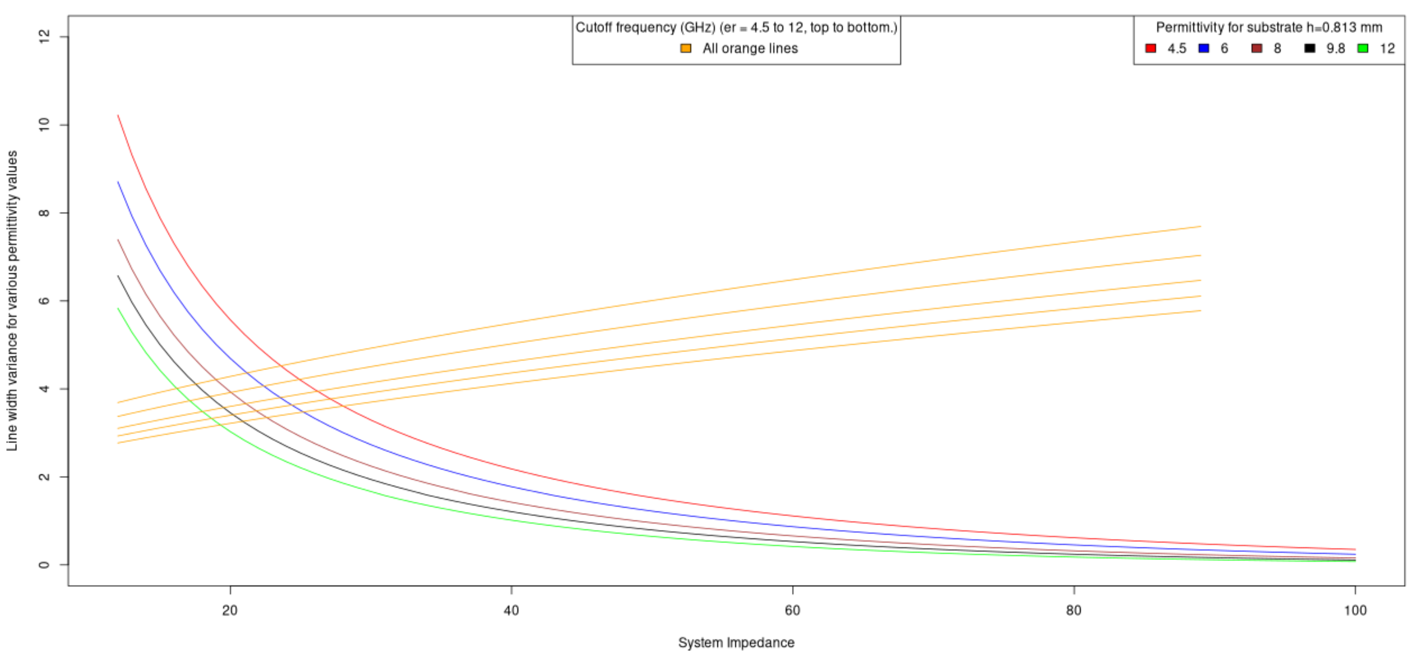


Figure 5: Line width variances for h=0.813

Also note the width variances decrease as the impedance becomes higher, regardless of board thickness. Comparing the two charts readily shows that the cutoff frequency, f_o, below which dispersion can be ignored, is lower for thicker substrates. However, the cutoff frequency is slightly higher for lower e_r values, which is rather non-intuitive.

These charts are generated using (Allaire et al. 2022) and (R Core Team 2022). I attempted to use ggplot2 to generate the charts, but it only accepts the *data.frame* format as input, whereas I placed my data in an *array* format, because the array has multiple dimensions (x,y,z (or slices)). When I converted it into a data frame, the data was scrambled. So rather than redesign the data storage format, I just used the standard *plot()* function. Then, by adding *lines()* to the basic plot, I created the charts you see here.

5. Vector network analysis.

Now I want to show analysis with a simple microwave filter. Firstly, I introduce the test equipment I am using to obtain these readings, the NanoVNA V2 Plus4 vector network analyzer. There are many clones in the wild that work okay, but after much research, I decided to acquire this particular model. Shielding and quality of components are important in a particular piece of test equipment. Otherwise the results may be subject to questioning or random interpretation.

Figure 6: NanoVNA V2 Plus4.

The full specifications for this particular model can be found on the nanorfe website[10]. The User

[10]NanoVNA specifications, https://nanorfe.com/nanovna-v2.html

Manual[11] and The Missing Manual[12] are both available on the website. The following are a select few of the specifications.

- Frequency range: 50kHz – 4.4GHz.
- Frequency resolution: 0.01MHz.
- Dynamic range: < -90 dB.
- Noise floor: -40 dB or less.

Calibration is fairly straightforward. Firstly, setting the frequency range of interest (called *STIMULUS*). Tapping the screen brings up the menu to select STIMULUS, START and STOP range (or CENTER and SPAN settings). For better resolution, select SWEEP POINTS and set 201. Next, select BACK, then CALIBRATE. Select RESET CAL, CALIBRATE, where a new menu appears to do OPEN, SHORT, LOAD, THRU. Then select DONE and BACK. SAVE the calibration on one of the slots.

On power up, you see this default screen, with frequency (bottom) set from 100 to 900 MHz. The top lines show the channels (CH0/CH1) where CH0 is Port 1 and CH1 is Port 2. The display is a bit cluttered, but shows some options available for output analysis.

Figure 7: NanoVNA default screen.

I will use both cables in this particular test, so will calibrate with the two cables connected. So, the OPEN, SHORT and LOAD are connected to the first cable on Port 1 (S11) using the SMA female-to-female 'barrel.' The THRU calibration uses the second cable on Port2 (S21) connected to the first cable. After

[11] User Manual: https://nanorfe.com/nanovna-v2-user-manual.html
[12] The Missing Manual, https://nanorfe.com/images/V2Plus4_Manual_NC4BR.pdf

connecting the DUT[13], under DISPLAY, TRACE, I turned off all traces except the first two (TRACE 0 AND TRACE 1). Port 1 (S11) is the output signal, and Port 2 (S21) shows the absorption through the DUT. I next adjusted the SCALE/DIV and REFERENCE POSITION settings (under SCALE) to place the traces where I desired. You have to first select the trace you wish to change, then set each appropriately.

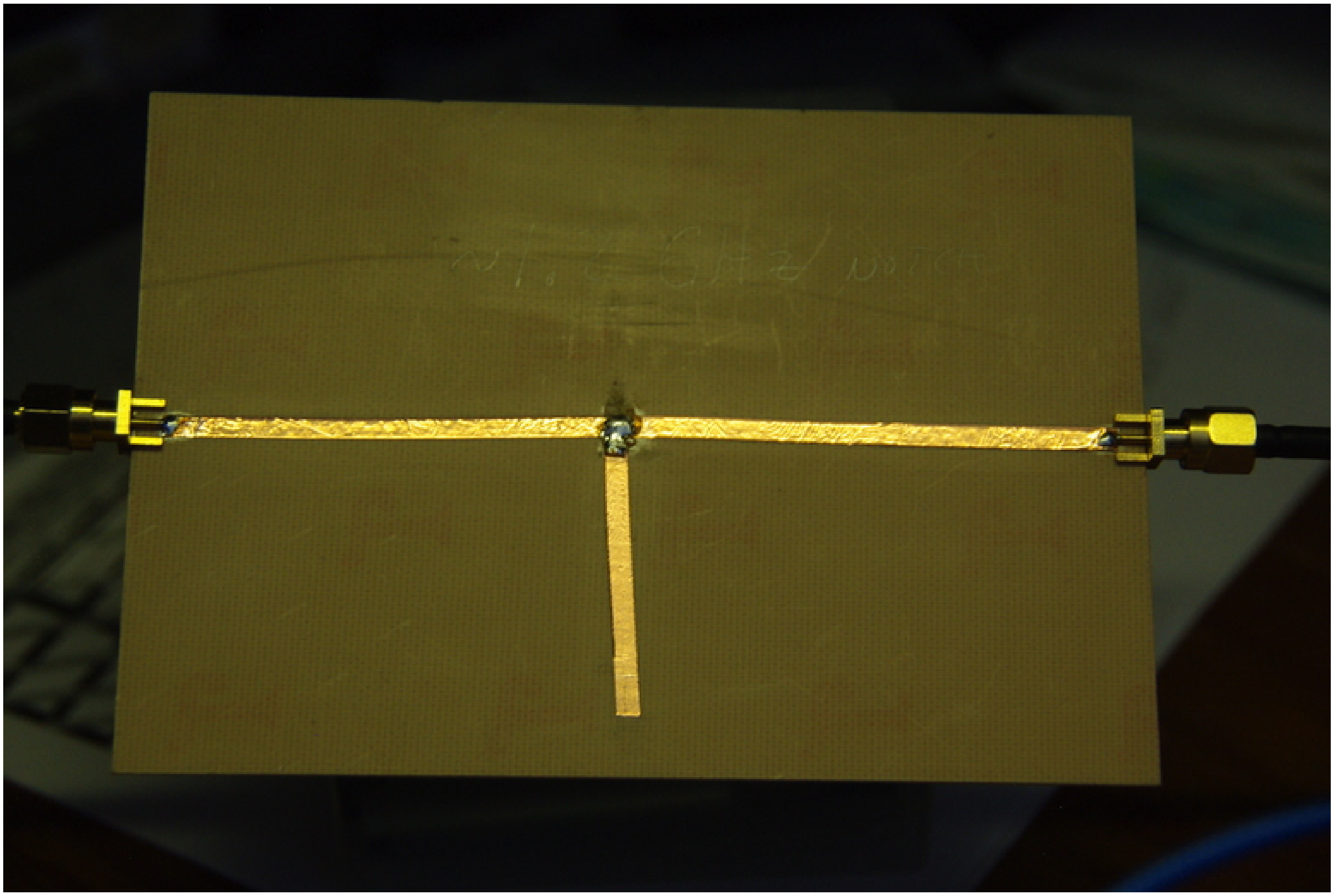


Figure 8: Filter layout.

The DUT is connected after calibration, using both cables. This particular circuit in on a single-sided PCB, and consists of stick-on copper tape, normally used for shielding (on guitars, for example.). The total copper thickness is 51 um, including the glue backing. So I suspect it is about 1 oz, or 35 um. The circuit was initially determined using (Wedge, Compton, and Rutledge 1991), then trimmed to tune it a bit. Nothing serious, just playing around for an example.

[13]Device under test.

Figure 9: Filter response.

After display adjustment, this is the response through the layout. Moving the marker to the dip shows -26.7 dBM at the tuned frequency of 1.2 GHz with both traces set to a logarithmic scale. I set the SCALE/DIV to 5 dBm per division for CH1, and 1 dBm for CH0. I also moved the REFERENCE POSITION to place CH1 in the best position (#6) for viewing(small blue arrow on left).

The copper tape is really handy for creating a circuit, as it can be trimmed easily. And, the results can be seen instantly on the NanoVNA. Great little tool for quick circuit design. I also checked it against my Tektronix 492P spectrum analyzer, and the results are very similar. The spectrum analyzer shows only amplitude, where the NanoVNA can also show phase. With the scale/div set to 14 and the reference set to 2, I get this display,

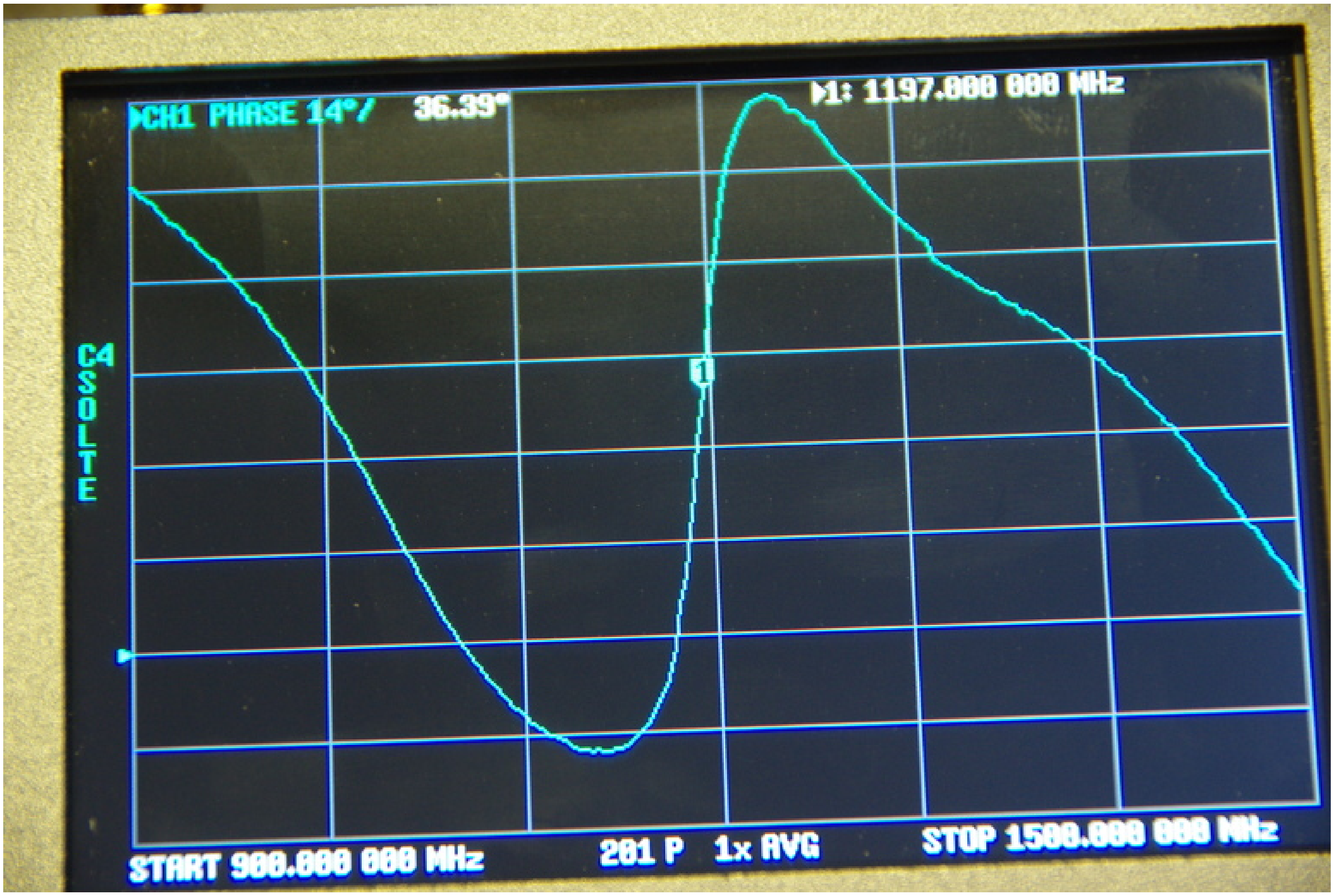


Figure 10: Filter phase.

This shows a phase angle of 36° at the marker, and depicts the change as the frequency changes. The below image depicting a Smith Chart, indicates the impedance is a bit higher than the ideal 50Ω, at 53.6Ω with 385 pH of inductance.

Figure 11: Smith chart.

As can be readily seen, the impedance is a bit on the inductive side, and the resistance is high. Some tuning could be done, such as trimming the parallel stub. However, if the copper traces were a bit wider, the impedance might be lower. Trimming the stub would move the frequency. What's nice with the copper tape is if I trim too much I can just cut another piece and replace the current one.

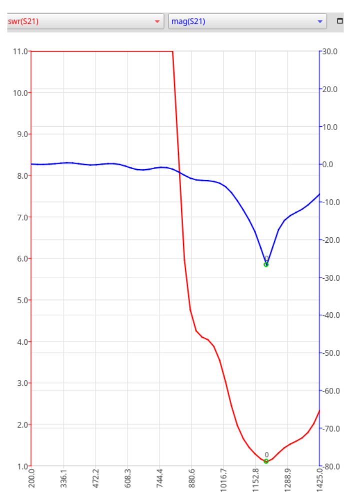


Figure 12: Showing Logmag and SWR.

Here, using the NanoVNA software, we have both the SWR and LogMag scales, color coded to match the traces. To see a change, I will trim the stub and see what happens.

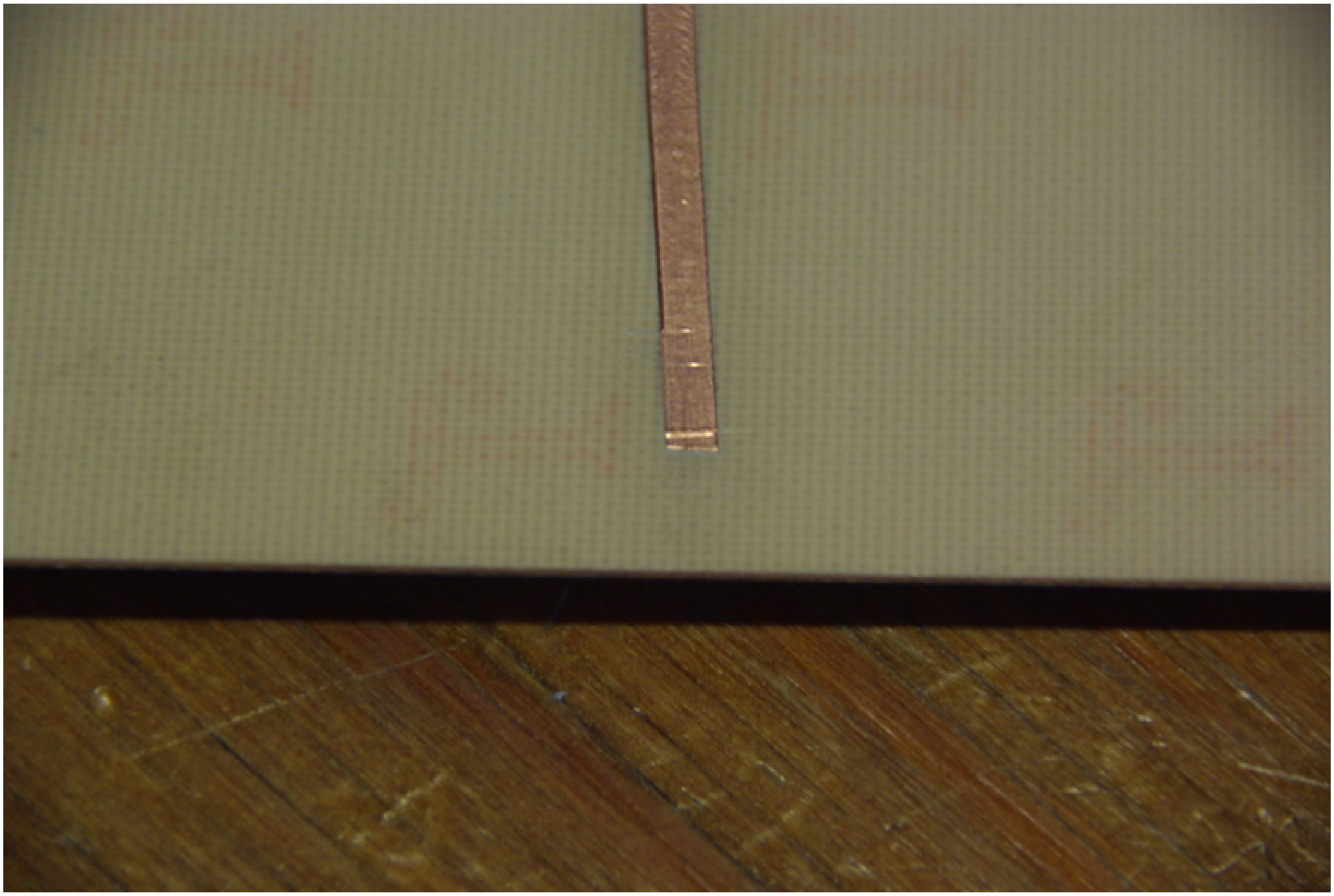


Figure 13: 1 mm trimmed.

The trimmed portion was about one mm. Notice that with just this little bit trimmed off the parallel open stub, the resonant frequency shifted upwards. I didn't move the below marker, so the shift can be easily seen.

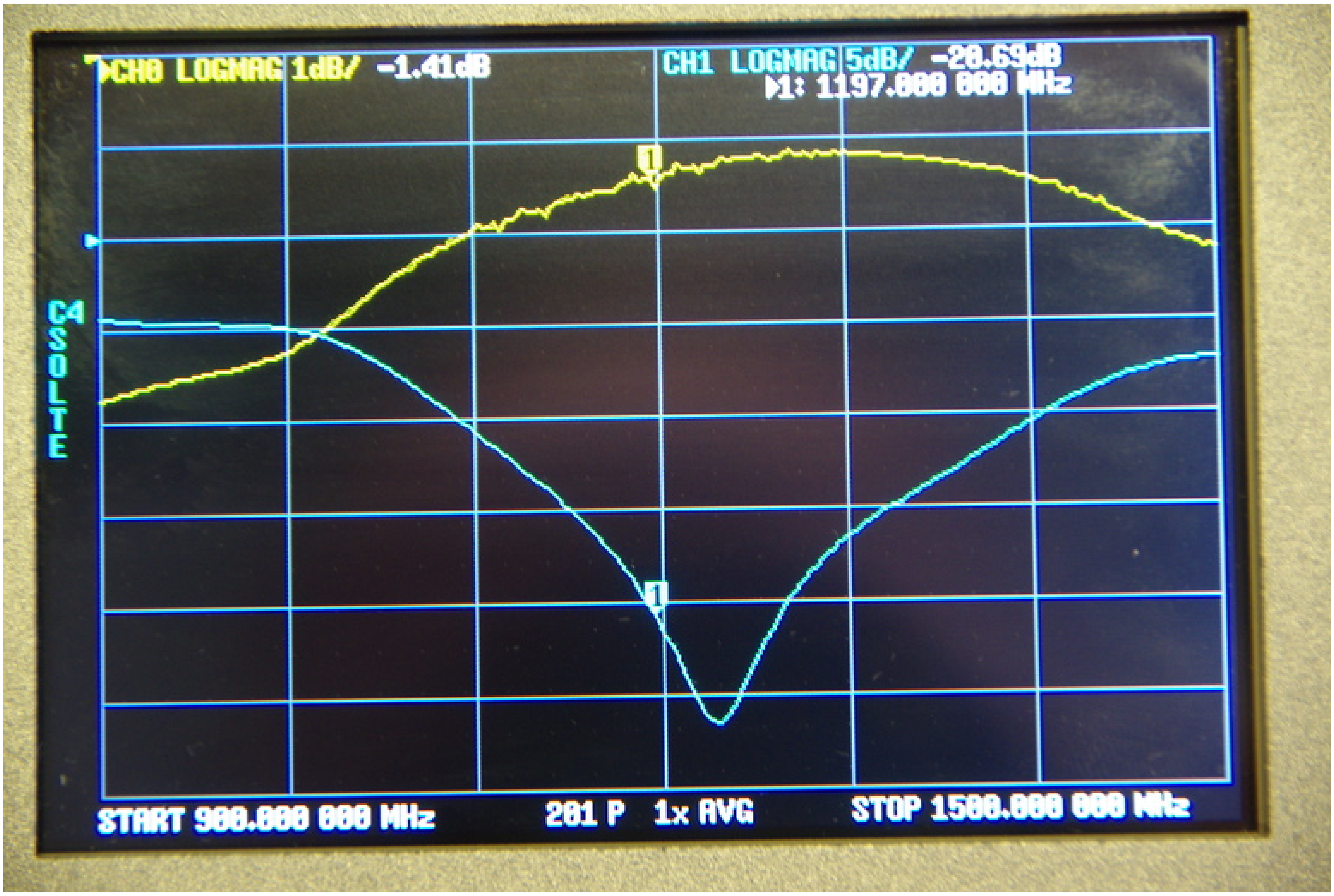


Figure 14: New response.

A shift to 1230 MHz is seen in the response, with about -26 dBm dip. The Smith Chart shows 54.5Ω with 270 pH inductance, compared with the former values of 53.6Ω with 385 pH of inductance.

This was just a quick introduction indicating the ease with which a filter can be tuned, using the NanoVNA.

Bandpass filter testing. Now I will build a bandpass filter (BPF) for the GPS[14] band, at 1575 MHz. Using a little R[15] program, I calculated the normalized values for a coupled line BPF with a degree of filtration (N) of three, a ripple factor of 0.0436 dB. I will use standard single-sided FR4 board, with a thickness of 1.57 mm, with the dielectric constant, ε_r of 4.5. Using a system impedance of 50Ω, I get these values,

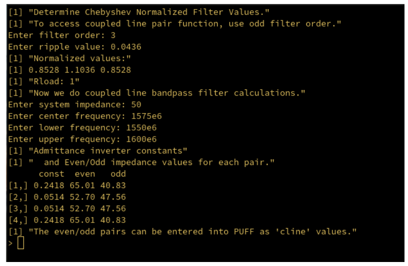


Figure 15: Coupled pair impedances.

The admittance inverter constants are an intermediate step to determine the impedance values for each pair of coupled lines. See Para. 3 for more details.

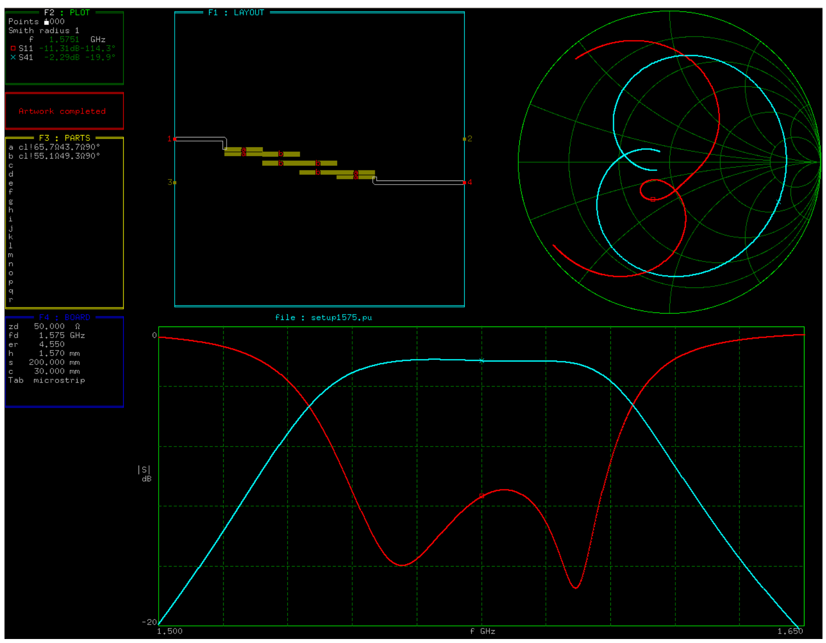


Figure 16: PUFF screen showing response curve.

The values were adjusted a bit to bring the real values to the input needed. The ‘cl!’ changes the values to compensate for various effects, and needs to be brought back to the desired inputs.

[14] Global Positioning System.
[15] (R Core Team 2022)

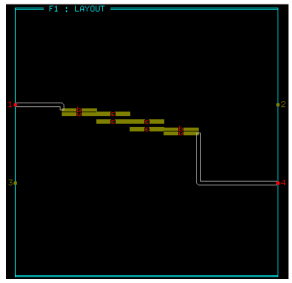


Figure 17: PUFF layout diagram.

After everything is optimized, the diagram looks like this. This particular layout size is 200 mm, which is a bit larger than I like to experiment with, so I may see if I can trim the ends of the diagram to make it fit on a 150 mm board. After printing the diagram, I will see if I can duplicate it using copper tape. Placing a strip of copper foil, then taping the printout on top worked okay. Using a sharp knife, I then traced the diagram, cutting through the foil. The final step was peeling the excess off.

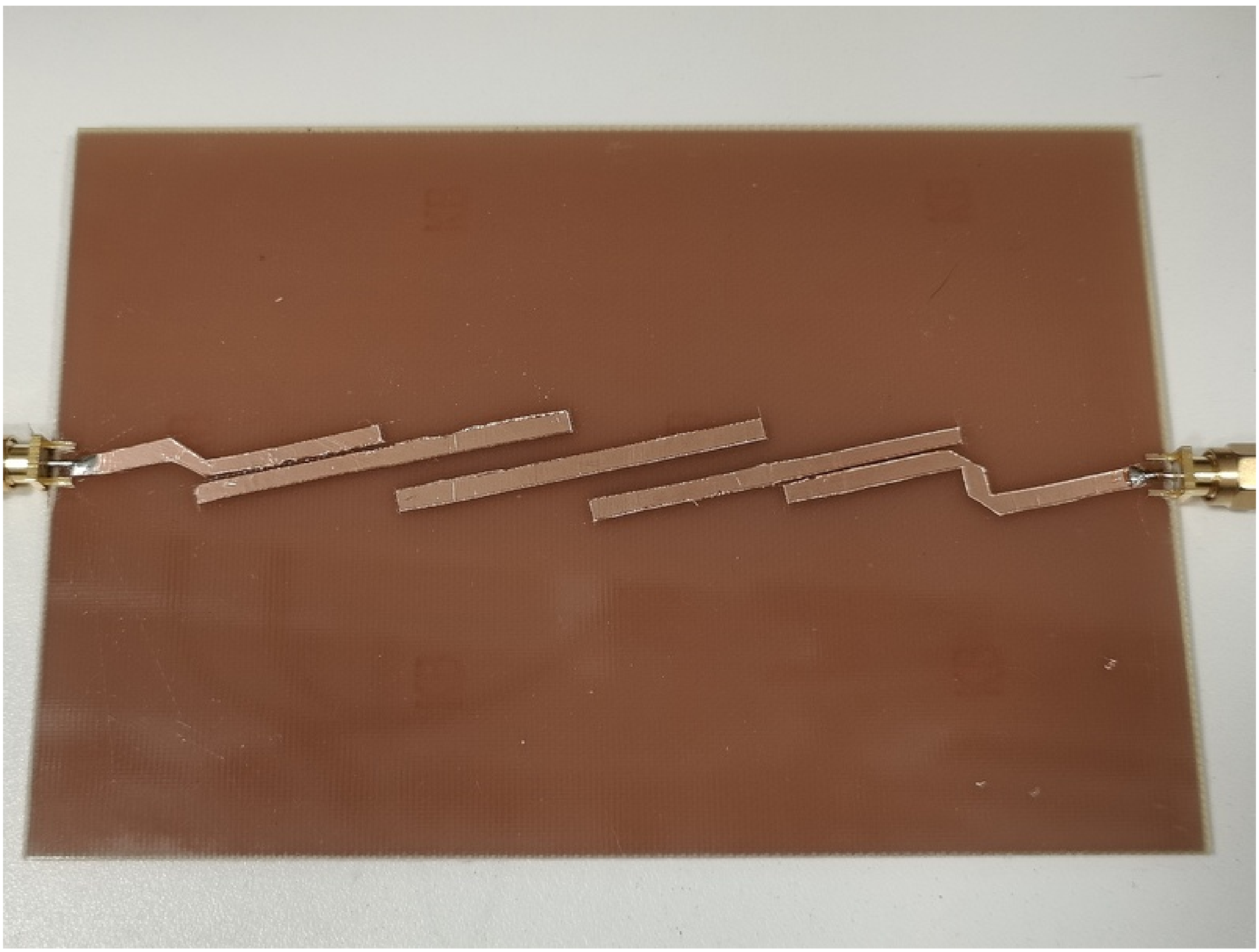


Figure 18: Filter on FR4.

Duplicating the circuit was fairly easy. However, getting the ends trimmed to account for dispersion was a bit harder. The value I wanted to remove from each end was about 0.18 mm. Controlling the knife was a bit more difficult, and I ended up cutting off more than I intended.

Figure 19: Frequency response.

As can be readily seen, I trimmed a bit too much, so the circuit tuned ad 1590 MHz instead of the desired 1575 MHz. That was the operator, not the circuit. Also the edges came out a bit ragged, cutting the copper foil. Overall, however, the concept is realizable for lower frequencies. Smaller sizes and higher frequencies would be harder to cut with accuracy. The return loss was a bit higher than it should be, probably because of the ragged edges. A sharper knife would have been better.

6. Further dispersion investigations.

Continuing with the formulae found in Chapter 7 of (Edwards and Steer 2016), defining improved expressions for dispersion at higher frequencies, the design formula by Kirschning and Jansen[16] covers a much wider range of permittivities, aspect ratios and frequencies.

Leading up to the main $\varepsilon_{eff}(f)$ expression, we have to present several preliminary formulae, where

[16] M. Kirschning and R. Jansen, "Accurate model for effective dielectric constant of microstrip with validity up to millimeter-wave freuencies." Electronics Letters, vol. 18, no. 6, pp. 272-273, 1982.

frequency, F, is in GHz, and thickness, H, is in cm. The first four are,

$$
\begin{aligned}
P_1 &= 0.27488 + [0.6315 + 0.525/(1 + 0.157FH)^{20}](w/h) \\
&\quad - 0.065683exp(-8.7513w/h) \\
P_2 &= 0.33622[1 - exp(-0.03442\varepsilon_r)] \\
P_3 &= 0.0363exp(-4.6w/h)(1 - exp[-(FH/3.87)^{4.97}]) \\
P_4 &= 1 + 2.751(1 - exp[-(\varepsilon_r/15.916)^8])
\end{aligned}
$$

These are used as input to this formula,

$$P(F) = P_1 P_2[(0.1844 + P_3 P_4)10FH]^{1.5763} \tag{41}$$

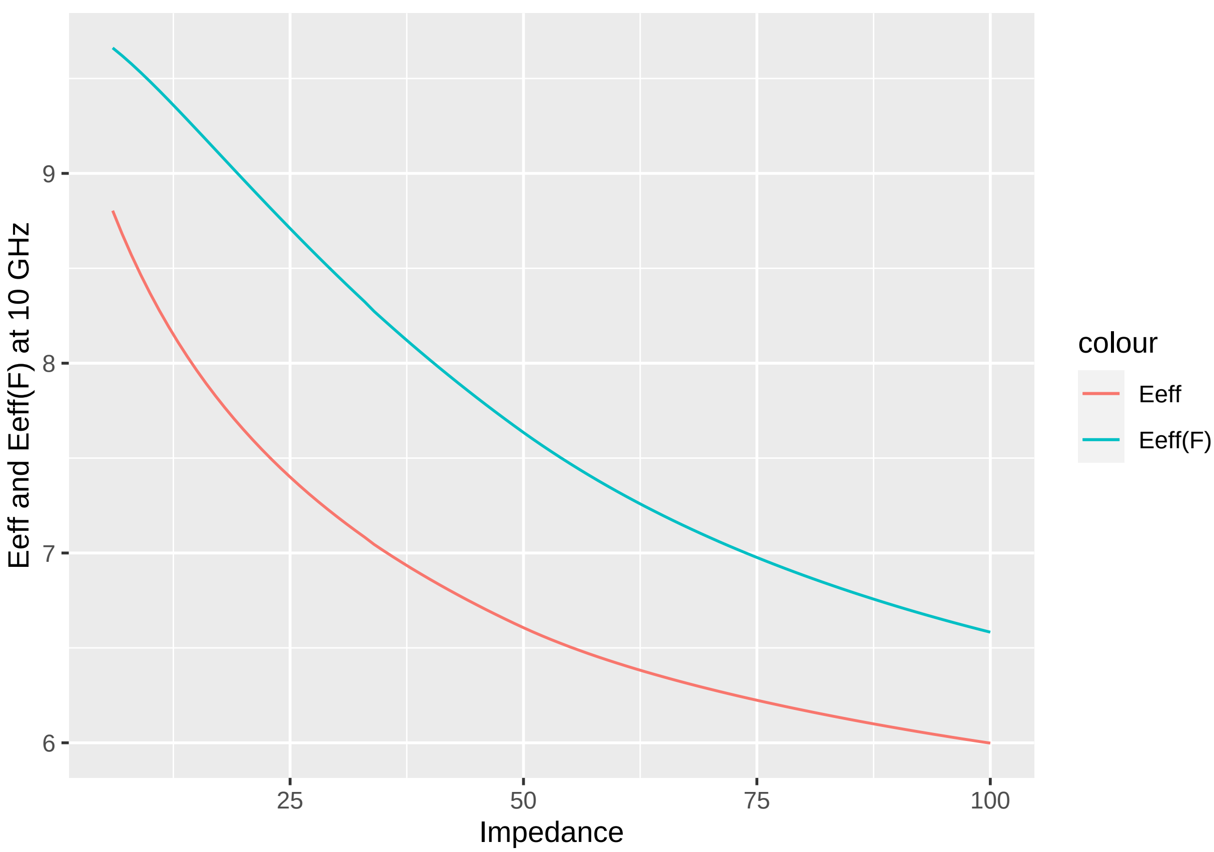
Finally we get to the basic formula,

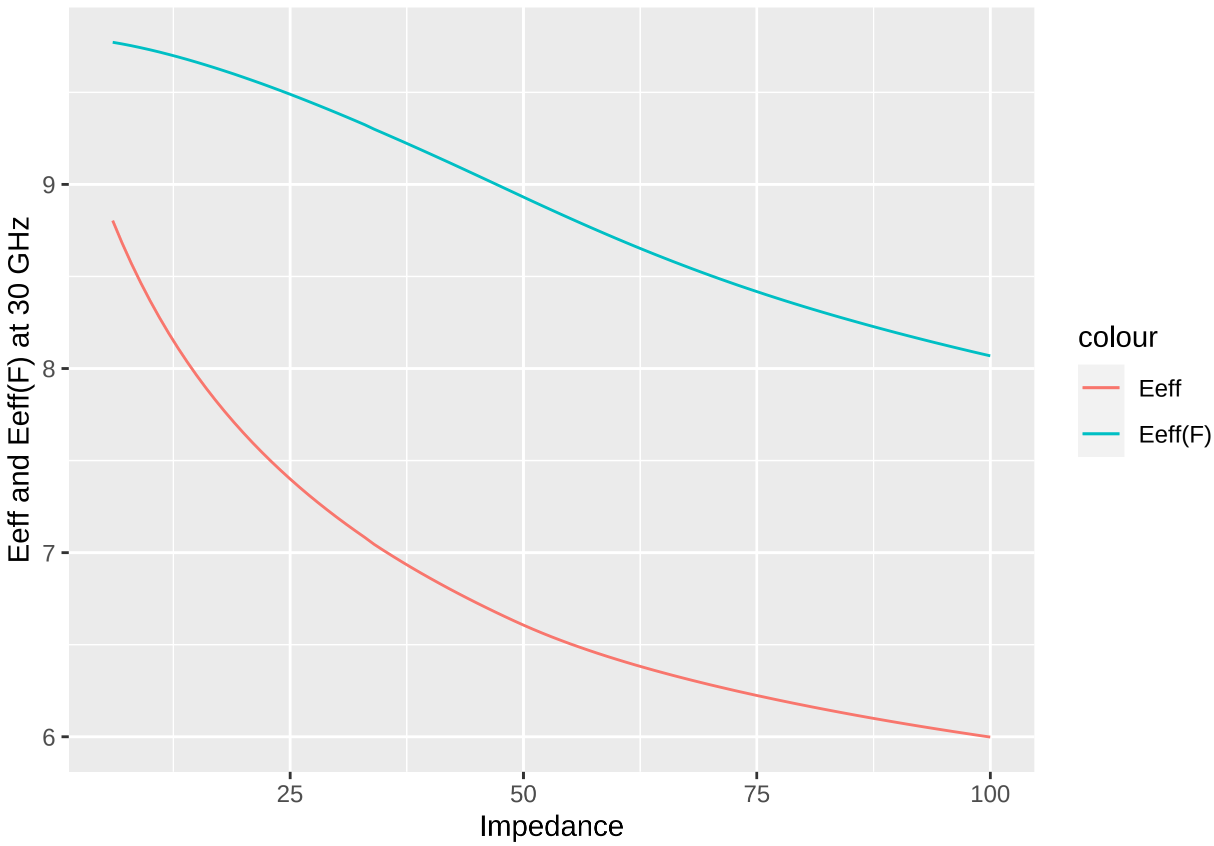
$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}}{1 + P(F)} \tag{42}$$

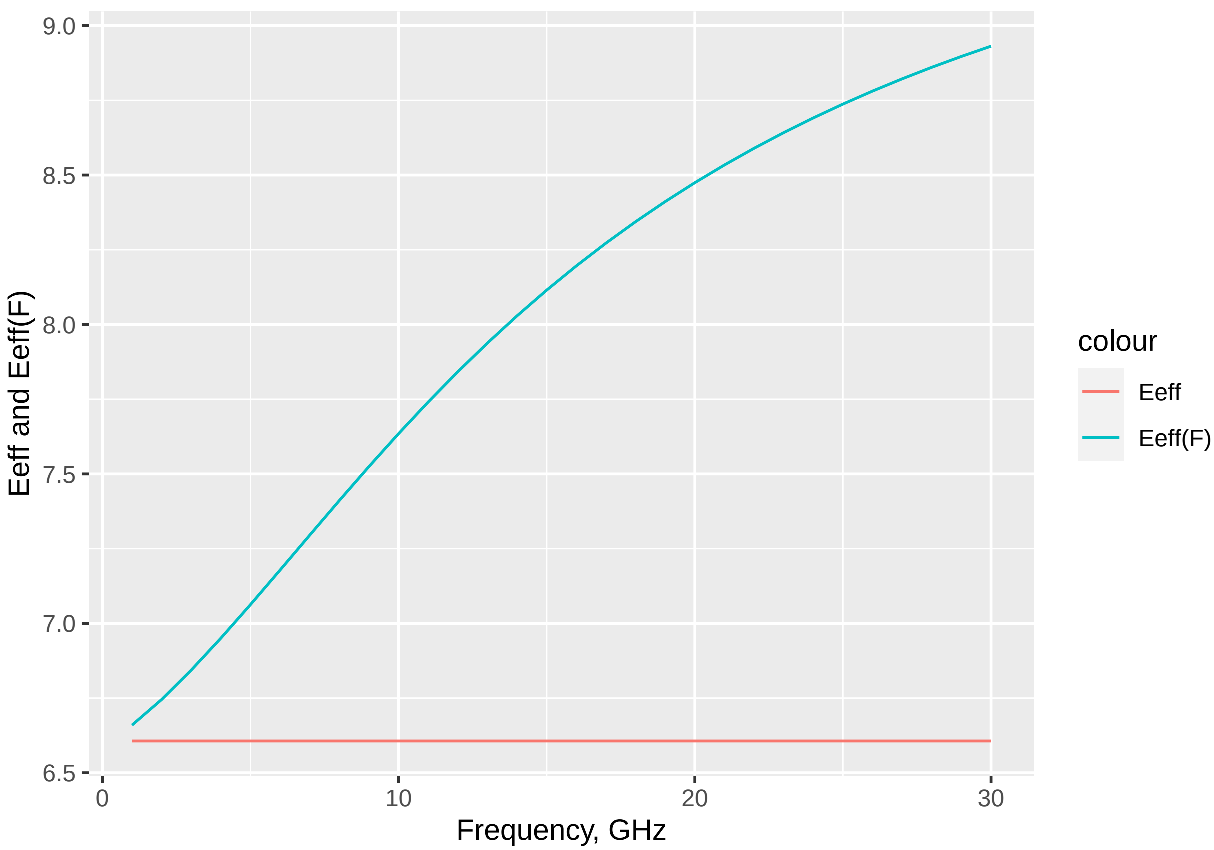
An accuracy of better than 0.6% is claimed to 60 GHz, but only tested to 30 GHz. The validity ranges are,

$$
\begin{aligned}
1 \le \varepsilon_r \le 20 \\
0.1 \le w/h \le 100 \\
0 \le h/\lambda_o \le 0.13
\end{aligned}
$$

So, what does that really look like? The below charts use a relative permittivity, $\varepsilon_r = 9.8$ and substrate, h=1.6. The deviation/difference over an impedance range is shown here, for 10 GHz and 30 GHz.







The ε_{eff} and $\varepsilon_{eff}(f)$ are mostly impedance driven, more so than frequency. However, when frequency goes higher, the $\varepsilon_{eff}(f)$ value rises, showing the dispersion effects at the higher frequencies; and at very high frequencies, where $F_r \geq 3000$ GHz, approaches ε_r.

That about wraps up these thoughts on dispersion effects at higher frequencies, and covers the areas I have been exploring. These ruminations are a great way to have the information available as I push on in my investigations of the microwave region and filter design in microstrip.

Allaire, JJ, Yihui Xie, Jonathan McPherson, Javier Luraschi, Kevin Ushey, Aron Atkins, Hadley Wickham, Joe Cheng, Winston Chang, and Richard Iannone. 2022. *Rmarkdown: Dynamic Documents for r*. https://CRAN.R-project.org/package=rmarkdown.

ARRL. 2023. *ARRL Handbook for Radio Communications, 100th Edition*. Newington, CT: American Radio Relay League. https://www.arrl.org.

Bahl, I. J., and D. K. Trivedi. 1977. *A Designer's Guide to Microstrip Line*. Microwaves, 1977-05 Vol 16 Issue 5. Penton Media, Inc.

Edwards, Terry C., and Michael B. Steer. 2016. *Foundations for Microstrip Circuit Design, 4th Edition*. West Sussex, United Kingdom: Wiley & Sons, Ltd.

Fink, Donald G., and Donald Christiansen. 1982. *Electronics Engineers' Handbook, 2nd Edition*. McGraw-Hill Book Company.

Hammerstad, E. O., and F. Bekkadal. 1975. *Microstrip Handbook*. ELAB Report. Norwegian Institute of Technology.

Pozar, David M. 2012. *Microwave Engineering, 4th Edition*. West Sussex, United Kingdom: Wiley & Sons, Ltd.

R Core Team. 2022. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. https://www.R-project.org/.

Wedge, Scott W., Richard Compton, and David Rutledge. 1991. *PUFF Computer Aided Design for Microwave Integrated Circuits, Version 2.0*. Pasadena, CA: California Institute of Technology.

Appendix A

I came to realize that there was a section missing from one of my favorite reference books, (ARRL 2023). The latest version I acquired, the 100th edition, no longer contains a section that depicted filter normalization value tables for filter design. The rationale may have been size limitations, or the fact many folks nowadays use some sort of software program to generate the values as needed. I'm not exactly sure when those references were removed as I don't have a handbook for every year, but they were removed sometime between the 2016 and 2021 editions. Admittedly, there are many other books available, mostly older volumes (read 20th Century) that contain pieces of those, or similar, tables. For example, Table 12-13 in (Fink and Christiansen 1982). However, the table layout seemed a bit easier to peruse in the ARRL Handbook than other references I've found.

As most filter orders (N) are not useful below 3, and too cumbersome above 11, those are the orders to which I limited my tables. The two filter types I create here are for Butterworth and Chebyshev designs.

Butterworth Filters As Butterworth filters have maximally flat responses in the passband, I didn't have to deal with ripple values as I do in Chebyshev filter tables. The formula to generate a Butterworth normalized value is:

$$g_x = 2\sin\left[(2 * N_x - 1)\frac{\pi}{2 * N_x}\right]$$

where $\mathrm{N_x}$ is the filter order, and $\mathrm{g_x}$ is the normalized value based on a radian corner frequency of 1 rad/s and a 1Ω system impedance. That means the angular frequency of oscillation is $\omega = 2\pi/T$ where T = time. The frequency of oscillation is 1/T. So then $\omega = 2\pi f$.

Chebyshev Filters The formulae are a bit more complicated for the Chebyshev design, but I will attempt to make them as clear as I can.

$$\gamma = \sinh\left(\frac{\beta}{2n}\right) \quad and \quad b_k = \gamma^2 + \sin^2\left(\frac{\beta\pi}{n}\right)$$

where k = 1,2,. . . ,n.

$$\beta = ln\left[\frac{\cosh(R_{dB}/2 * 20\log(e))}{\sinh(R_{dB}/2 * 20\log(e))}\right] \quad = \quad ln\left[\coth\left(\frac{R_{dB}}{17.3717793}\right)\right]$$

where e is 2.718281828, $\mathrm{R_{dB}}$ is ripple, n is filter order, γ is Gamma, β is Beta. For the normalized values g_k, we use:

$$a_k = \sin\left[\frac{(2k-1)\pi}{2n}\right] \quad for \quad g_1 = \frac{2a_1}{\gamma}$$

so then:

$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}$$

Finally, the R_{load} value for odd orders is 1, for even orders:

$$g_{n+1} = \tanh^2\left(\frac{\beta}{4}\right)$$

Obviously, it is generally more desirable to have source and load impedances the same, so Chebyshev filters are usually restricted to odd orders.

Table 1: Butterworth Normalized Filter Values

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	Rload
N(3)	1.000	2.000	1.000	NA	NA	NA	NA	NA	NA	NA	NA	1
N(4)	0.765	1.848	1.848	0.765	NA	NA	NA	NA	NA	NA	NA	1
N(5)	0.618	1.618	2.000	1.618	0.618	NA	NA	NA	NA	NA	NA	1
N(6)	0.518	1.414	1.932	1.932	1.414	0.518	NA	NA	NA	NA	NA	1
N(7)	0.445	1.247	1.802	2.000	1.802	1.247	0.445	NA	NA	NA	NA	1
N(8)	0.390	1.111	1.663	1.962	1.962	1.663	1.111	0.390	NA	NA	NA	1
N(9)	0.347	1.000	1.532	1.879	2.000	1.879	1.532	1.000	0.347	NA	NA	1
N(10)	0.313	0.908	1.414	1.782	1.975	1.975	1.782	1.414	0.908	0.313	NA	1
N(11)	0.285	0.831	1.310	1.683	1.919	2.000	1.919	1.683	1.310	0.831	0.285	1

Table 2: Chebyshev Normalized Filter Values, 0.01 R_{dB}.

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	Rload
N(3)	0.629	0.970	0.629	NA	NA	NA	NA	NA	NA	NA	NA	1.000
N(4)	0.713	1.200	1.321	0.648	NA	NA	NA	NA	NA	NA	NA	0.908
N(5)	0.756	1.305	1.577	1.305	0.756	NA	NA	NA	NA	NA	NA	1.000
N(6)	0.781	1.360	1.690	1.535	1.497	0.710	NA	NA	NA	NA	NA	0.908
N(7)	0.797	1.392	1.748	1.633	1.748	1.392	0.797	NA	NA	NA	NA	1.000
N(8)	0.807	1.413	1.782	1.683	1.853	1.619	1.555	0.733	NA	NA	NA	0.908
N(9)	0.814	1.427	1.804	1.713	1.906	1.713	1.804	1.427	0.814	NA	NA	1.000
N(10)	0.820	1.437	1.819	1.731	1.936	1.759	1.905	1.653	1.582	0.745	NA	0.908
N(11)	0.823	1.444	1.830	1.744	1.955	1.786	1.955	1.744	1.830	1.444	0.823	1.000

Table 3: Chebyshev Normalized Filter Values, 0.044 R_{dB}.

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	Rload
N(3)	0.854	1.104	0.854	NA	NA	NA	NA	NA	NA	NA	NA	1.000
N(4)	0.934	1.293	1.581	0.764	NA	NA	NA	NA	NA	NA	NA	0.818
N(5)	0.974	1.372	1.804	1.372	0.974	NA	NA	NA	NA	NA	NA	1.000
N(6)	0.997	1.413	1.896	1.550	1.728	0.815	NA	NA	NA	NA	NA	0.818
N(7)	1.011	1.437	1.942	1.621	1.942	1.437	1.011	NA	NA	NA	NA	1.000
N(8)	1.020	1.452	1.969	1.657	2.026	1.610	1.775	0.834	NA	NA	NA	0.818
N(9)	1.026	1.462	1.986	1.677	2.067	1.677	1.986	1.462	1.026	NA	NA	1.000
N(10)	1.031	1.469	1.998	1.690	2.090	1.709	2.066	1.633	1.796	0.843	NA	0.818
N(11)	1.034	1.474	2.006	1.698	2.105	1.727	2.105	1.698	2.006	1.474	1.034	1.000

Table 4: Chebyshev Normalized Filter Values, 0.2 R_{dB}.

	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	Rload
N(3)	1.224	1.153	1.224	NA	NA	NA	NA	NA	NA	NA	NA	1.000
N(4)	1.299	1.285	1.972	0.847	NA	NA	NA	NA	NA	NA	NA	0.652
N(5)	1.336	1.338	2.162	1.338	1.336	NA	NA	NA	NA	NA	NA	1.000
N(6)	1.356	1.364	2.236	1.457	2.094	0.884	NA	NA	NA	NA	NA	0.652
N(7)	1.369	1.379	2.272	1.502	2.272	1.379	1.369	NA	NA	NA	NA	1.000
N(8)	1.377	1.389	2.293	1.523	2.338	1.494	2.131	0.897	NA	NA	NA	0.652
N(9)	1.382	1.395	2.306	1.536	2.370	1.536	2.306	1.395	1.382	NA	NA	1.000
N(10)	1.386	1.399	2.315	1.543	2.387	1.555	2.369	1.508	2.148	0.903	NA	0.652
N(11)	1.389	1.403	2.321	1.549	2.398	1.566	2.398	1.549	2.321	1.403	1.389	1.000