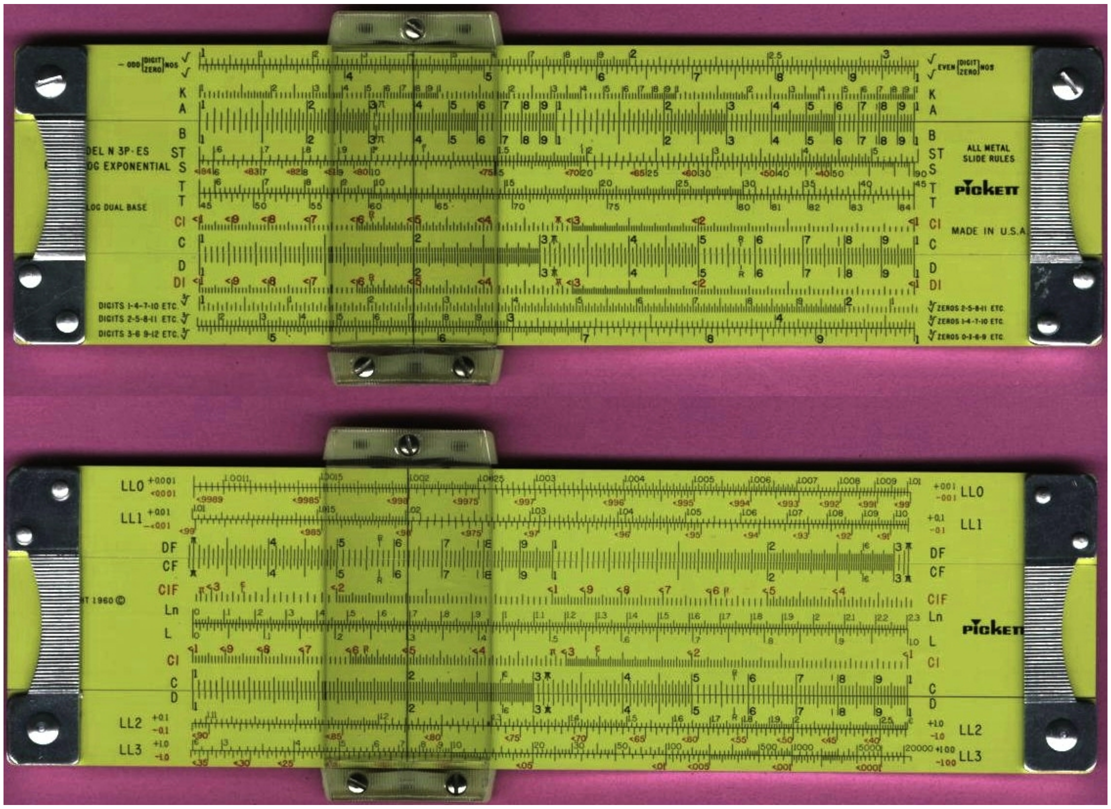

Figure 1: Pickett N3P-ES Slide Rule.

Slide Rule Instructions

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Table of contents

Introduction

Slide Rule Scales.

How the scales work. Using the N4-ES slide rule as an example, which has 34 scales, the scales are two C][D, CF][DF, two CI, CIF, DI, T, S, ST, two $\sqrt{}$ scales, three $\sqrt[3]{}$ scales, four LL and four -LL scales, DF/$_{\mathrm{M}}$][CF/$_{\mathrm{M}}$, two TH, two SH, Ln, and L. The '][' symbol means one side is on the slide, the other on the stock (or stator).

C][D scales: x**,**. the C and D scales, C on the slide, D on the body.

CF][DF scales: $\pi \times x$. The CF][DF scales are C][D scales folded at π used primarily to reduce cases where the slide may project far out one side or the other, and to reduce having to reset the slide.

CI][DI scales: $\frac{1}{x}$. The CI][DI scales are reversed C][D scales used for finding reciprocals. The CI scale assists in multiplication or division where the slide may project too far out one side similar to the CF][DF scales, and performs like the C][D scale, only the motions are reversed (i.e., C over D results in division, whereas CI over D results in multiplication; and where C index at one factor and HL at second factor results in product on D scale (multiplication), the same with CI index results in the quotient on D scale (division).

CIF scale: $\frac{1}{(x \times \pi)}$. The **CIF** scale is a folded CI scale.

CF/$_{\mathbf{M}}$][DF/$_{\mathbf{M}}$ scales. These scales are folded at 1/M=2.3 where modulus M=$\log_{10} e$. The CF/$_{\mathrm{M}}$][DF/$_{\mathrm{M}}$ scales directly indicate Ln of the LL scales' reading, and D scale shows the $\log_{10}$ value. They can also be used the same as the C][D scales for normal operations.

A][B and K scales. The A][B and K scales are replaced on the N4 with separate $\sqrt{}$ and $\sqrt[3]{}$ scales respectively. However, the 600 has A/B scales, as does the K&E Deci-Lon.

T, S, ST scales. These scales are for tangents/cotangents, sines/cosines, and the reciprocal functions, and show the results on the C or D scales. The ST scales is used for sines or tangents $< 5.7^{o}$. The S scale shows sines left to right, and cosines right to left.

SH, TH scales. The hyperbolic sine and hyperbolic tangent scales determine results for electrical engineering computations (i.e., $\sinh x, \cosh x, \tanh x$).

L, Ln scales. The Ln scale show the mantissa of a number to several digits of precision, and adding the characteristic allows finding any power of *e*. The L scale directly reads $log_{10}x$ where x $<$ 10. Larger numbers are computed.

Decimal Point Location.

When counting the number of digits (or span), decimal zeros are negative. For example, the below chart illustrates this concept.[1]

Number	Span or Digit	Number	Span or Digit
3520	4	0.352	0
352.0	3	0.0352	-1
35.2	2	0.00352	-2
3.52	1	0.000352	-3
0.352	0	0.0000352	-4

Multiplication with the C and D scales.

- If the slide projects to the right of the stock, the digit count for the product is one less than the sum of the digit counts for the multiplicand and multiplier.
- If the slide projects to the left of the stock, the digit count for the product is equal to the sum of the digit counts for the multiplicand and multiplier.

Multiplication with the CI and D scales.

- If the slide projects to the right of the stock, the digit count for the product is equal to the sum of the digit counts for the multiplicand and multiplier.
- If the slide projects to the left of the stock, the digit count for the product is one less than the sum of the digit counts for the multiplicand and multiplier.

Division with the C and D scales.

- If the slide projects to the right of the stock, the digit count for the quotient is one more than the digit count for the dividend minus the digit count for the divisor.
- If the slide projects to the left of the stock, the digit count for the quotient is equal to the digit count for the dividend minus the digit count for the divisor.

Division with the CI and D scales.

- If the slide projects to the right of the stock, the digit count for the quotient is equal to the digit count for the dividend minus the digit count for the divisor.
- If the slide projects to the left of the stock, the digit count for the quotient is one more than the digit count for the dividend minus the digit count for the divisor.

An easy way to remember the above rules, using the C and D scales, is this chart, for the left and right ends respectively.[2] Reverse the below if using with CI, CIF or DIF scales.

Left	Projection	[]	Right	Projection
×	SUM	[]	×	SUM -1
÷	DIFF	[]	÷	DIFF +1

[1] See “Slide Rule, How to use it, Third Edition” by Calvin C. Bishop, 1955, Barnes & Noble.
[2] See “Dimensional Analysis and Conversions” for signed numbers rules.

It may be simpler if the calculation digits are figured first, then the slide projections added. Add positive digits, then subtract negative digits: 1 + (-3) + 4 is 5 - 3 = 2.

Special Cases. When the same product may be obtained with the slide projecting either to the left or the right, use the left-handed projection in finding the place for the decimal point.
When the rule closes up in making computations, call the setting that makes the rule close up a right-handed projection.

Example: $\frac{235\times0.00047\times0.15_R}{0.062\times34\times0.0000075_R}$ If cross-multiplying, digits are: 3 - (-1) + (-3) - 2 -(-5) = 4 digits. The zero for 0.15 is not included.[3] Next, during calculation the slide projects right twice as shown by the subscript R, +1 and -1, which cancel. The final answer is 1048.
Example: $\frac{755}{0.0272} = \frac{7.55_E{}^2}{2.72_E{}^{-2}}$. The slide projects to the right, so we add 1 to difference: 3 - (-1) + 1 = 5. So the quotient of 278 is 27800. Using scientific notation, the 2 - (-2) = 4, so 2.78 becomes $2.78_E{}^4$.
Example: $\frac{0.0000215}{24600}$ The digits (span) are (-4) - 5 = -9 and the slide is left. So 0.875 becomes 0.000000000875, or $8.75_E{}^{-10}$.
Example: .023 × 14. the digits are (-1) + 2 -1 = 0. So 322 becomes 0.322.

For combined operations, note the slide end's direction during each calculation to add or subtract digits (or span) for final answer.[4]

Example: $\frac{6\times0.35}{81\times0.005} = 52$. Set 81 (C scale) over 6 (D scale), move HL to 3.5 (C scale), move slide to place 5 (C scale) under HL. Read 5185 (D scale) at index. Digits for each operation (slide all to left end) are: (-1) +0 - (-1) = 1, giving the result of 5.185.
Example: $\frac{0.015\times4}{2.5\times1.2} = \frac{0.06}{3}$ Set 2.5 (C scale) over 1.5 (D scale) (slide left), slide HL to 4 (C scale), move slide to place 1.2 (C scale) (slide right) under HL, read 2 at index (D scale). Digit placement: (-1) -1 +1 -1 = -2, so final answer is 0.02.

Scientific Notation.

Scientific notation is where a number is placed in the form $N_{10}{}^x$ so that $1 < N < 10$. For example, 2547 is written as $2.547_{10}{}^3$. This form can assist in decimal placement of a derived answer.

Example: If a calculation of 234.7 x 0.015 is shown as $2.347_{10}{}^2$ x $1.5_{10}{}^{-2}$ giving 3.52, the decimal is found by $x^2 + x^{-2} = 0$, so the correct answer is 3.52.
Example: Where 0.0165 / 23 becomes $\frac{1.65_{10}{}^{-2}}{2.3_{10}{}^{1}}$,the (-2) -1 is $.717_{10}{}^{-3}$, or 0.000717.
Example: For $\frac{0.000742}{124{,}500} = \frac{7.42_E{}^{-4}}{1.245_E{}^5} = 5.96_E{}^{-9}$.

The exponent rules are, for multiplication, add exponents; for division, subtract exponents.[5] It may be necessary to move the decimal to the right if multiplying by a power of 10 and to the left if dividing by a power of 10.

[3] Alternatively, the top and bottom could be done separately. 3 + (-3) = 0, (-1) + 2 + (-5) = -4; 0 - (-4) = 4 digits.
[4] See "Combined Multiplication and Division" section for procedure.
[5] See "Dimensional Analysis and Conversions" for signed numbers rules.

Operations

Multiplication.

$P = X \times Y$. Set the index of the C scale over one of the factors on the D scale. Move the hairline (HL) over the other factor on the C scale, and read the product under the HL on the D scale.

Example: Convert 23.9º Celsius to degrees Fahrenheit. Firstly, multiply 23.9 by 1.8 for 43 (D scale), then add 32 for 75º F.

Division.

$Q = \frac{X}{Y}$. Set the divisor (on the C scale) opposite the number to be divided (on the D scale). Read the result, or quotient, on the D scale under the index of the C scale.

Example: Convert 75º Fahrenheit to degrees Celsius. Subtract 32 from 75 then set 1.8 of C scale over 43 (D scale) to read 23.9º Celsius at index.

Continued Products.

$P = X \times Y \times Z$. Set the C index at X on the D scale. Move the HL over Y on the C scale. Move the C index under the HL. Move the HL over Z on the C scale. Continue moving HL and C index alternately until all numbers have been set. Read the product under the HL on the D scale. Quick check for correct operation "in your hands" is 5-4-5 sequence. Multiply any number $N \times 5 \times 4 \times 5$. You should end up at the starting number.

Combined Multiplication and Division.

$Q = \frac{X \times Y \times Z}{R \times S \times T}$. Move R (denominator) on the C scale over X (numerator) on the D scale. Set the HL over Y on the C scale. Move S on the C scale under the HL. Continue moving the HL and slide alternately until all numbers have been set. Read the result on the D scale. If there is one more factor in the numerator (top) than the denominator (bottom), the result is under the HL. If the number of factors in the numerator and denominator are the same, the result is under the C index. **Dividing by a number is the same as multiplying by its reciprocal**. If multiplication required, and no numerator, use reciprocal of denominator. If division required and no denominator, use reciprocal of numerator. Examples:

$$\frac{4.2 \times 83}{21_R \times 8^2 \times 14_R} = \frac{4.2 \times 83 \times \frac{1}{14}}{21 \times 8^2} = 0.0185$$

$$\frac{31 \times 24^R \times 14}{25} = \frac{31 \times 24}{25 \times \frac{1}{14}} = 417$$

The **R** in the above examples denotes when the slide projects to the right. Superscript is multiplication, subscript is division.

Proportion and Percentage.

$\frac{R}{S} = \frac{T}{x}$. Set R on C scale opposite S on D scale. Under T on C scale, read x on D scale. This function creates a table of any numbers set. Notice that when the C index is opposite 2 on the D scale, the ratio of 1:2 (50%, or 0.50) is set for all other opposite graduations. Add a percentage by multiplying by 1+% in decimal, i.e., 35 + 20% is 35 x 1.20.

Example: $\frac{7}{22} = \frac{13}{x}$. Set 7 (C scale) over 22 (D scale). Swap ends, or set HL to 13 on CF scale. Read 41 on DF scale.

Example: 20% of 80. Set index to 80 on D scale, HL to 20 on C scale. Read 16 on D scale. Multiplication: 0.20 x 80 = 16.

Example: 20 is 40% of x. Set 40 on C scale over 20 on D scale. Read 50 on D scale. Division: 20 / 0.40 = 50.

Example: A metal is 25 parts copper & 12 parts tin. Find ratios in 1850 total lbs. Baseline is total (25+12=37). $\frac{25}{37} = \frac{x}{1850}$ and $\frac{12}{37} = \frac{x}{1850}$. Set 25 (C scale) over 37 (D scale), read 1250 (C scale) over 1850 (D scale). Set 12 over 37, move slide, or use CF/DF scales and read 600 (CF scale) over 1850 (DF scale).

Example: 25 men do a job in 30 days. Time for 35 men? This is inverse, so set 25 (CI scale) over 30 (D scale). Move HL to 35 (CI scale), read 21.4 (D scale).

Determining heights using shadows: Measure shadow of a pole and shadow of object. Proportion: $\frac{pole_S}{pole_H} = \frac{obj_S}{obj_H}$.

Example: A tree has 115 ft shadow where an 8 ft pole has 5 ft shadow. Tree height? $\frac{5}{8} = \frac{115}{x}$. Set 5 (C scale) over 8 (D scale). Move HL to 115 (C scale), read x = 184 (D scale).

Areas and volumes: *Areas* of similar figures compare to the *squares* of corresponding dimensions. A, c scales: $\frac{A1}{c1} = \frac{A2}{c2}$

Volumes of similar figures compare to the *cubes* of corresponding dimensions. Using K, c scales: $\frac{K1}{c1} = \frac{K2}{c2}$

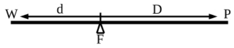
Example: A figure's dimension of 3.15 in. has a surface area of 4.84 in^2. A similar figure's dimension is 5.26 in. Surface area? Set HL to 4.84 (A scale), slide 3.15 (C scale) under HL. Move HL to 5.26 (C scale), read 13.5 in^2 on A scale.

Example: A figure's dimension of 3.15 in. has a volume of 15.6 in.3. A similar figure's dimension is 5.26 in. Volume? Set HL to 15.6 (K scale), slide 3.15 (C scale) under HL. Move HL to 5.26 (C scale), read 72.5 in.3 on K scale.

Example: Map scale drawing, 1:5000. Find acres for 168 in^2: $\frac{168}{1} = \frac{42}{5000}$. Set HL to 168 (A scale), slide index (C scale) under HL. Move HL to 5000 (C scale), read 42 on A scale. 5000^2=$25_\mathrm{E}6$. And $25_\mathrm{E}6(168) = 42_\mathrm{E}8$ in^2. Use dimensional units to convert to acres: $\frac{42_E 8 \cancel{in.^2}}{1}\left(\frac{1\cancel{ft.^2}}{144\cancel{in.^2}}\right)\left(\frac{1acre}{43560\cancel{ft.^2}}\right) =$ 670 acres.

Hydraulics:1 P1=P2 (Pascal's principle): $\frac{F_1}{A_1} = \frac{F_2}{A_2}$. P=pressure, A=area, F=force.

Example: 5 lbs pressure on 1.51 in^2 is ? pressure on 240 in^2. Set 5 (C scale) over 1.51 (D scale), read 795 over 240.

Levers and Fulcrums: $\frac{W}{d} = \frac{P}{D}$ 

This is an inverse ratio, so weight is set on CI scale. F = fulcrum, W = weight, P = pressure applied, d = distance to weight, D = distance to pressure.

Example: Weight 150 lb 10 ft from fulcrum, Pressure 3 ft from fulcrum. $\frac{150}{10} = \frac{P}{3}$. Set 150 (CI scale) over 10 (D scale). HL to 3 (D scale), read P = 500 (CI scale).

Transformer Z matching: $Z_P = Z_S\left(\frac{N_P}{N_S}\right)^2$, $Z_S = \frac{Z_P}{\left(\frac{N_P}{N_S}\right)^2}$, $N_P = N_S\sqrt{\frac{Z_P}{Z_S}}$, $N_S = \frac{N_P}{\sqrt{\frac{Z_P}{Z_S}}}$,

Ratio: $\frac{N_P}{N_S} = \sqrt{\frac{Z_P}{Z_S}} = \frac{V_P}{V_S} = \frac{I_S}{I_P}$.

Reciprocal of a Number.

$\frac{1}{N}$. When a number is set under the hairline (HL) on the C scale, its reciprocal is found under the HL on the CI scale. Decimal placement same as log characteristic plus 1 (For 10 or powers, do not add one).

Examples: $\frac{1}{20} = 0.05$, $\frac{1}{1000} = 0.001$

Radian Measure.

2π radians = 360°, 1 radian = 57.29578°.

Radians to Degrees: When the C index is set over any number of radians on the D scale, under R on the C scale read the corresponding number of degrees on the D scale.

Example: 5.2 radians = 298°. Example: 6 radians = 344°.

Degrees to radians: Set C scale “R” over degrees on D scale. Read radians at C index.

Example: 35° = 0.611 radians. Example: 171° = 2.98 radians. For small angles (in the ST scale range), HL to angle, find radians on C scale (0.01 to 0.1).

Example 2.4° = 0.0418 radians. Example: 0.62° = 0.0108 radians. **Radians per second to frequency in Hz**: $f_{Hz} = \frac{rad/sec}{2\pi}$

Circle Formulas.

Area of a Circle: πr^2, **Cylinder Volume**: $\pi r^2 h$
Method 1: Set the B index to ’, (0.7854) on the A scale. Move the HL over the diameter on the C scale. Read the area under the HL on the A scale. For volume, set B index on area value, HL on height (same B scale side as index) and read volume on A scale. When the area is known, reverse for diameter.
Method 2: Set radius on dual $\sqrt{}$ scale. Read area on DF scale. For volume, set CF index on area, HL to height (CF scale), read volume (DF scale).

Circumference of a Circle: πd (derivative of area). Set the HL over the diameter of the circle on the D scale. The circumference is under the HL on the DF scale.

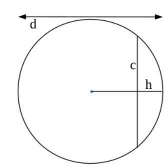
Length of Arc: $s = r\theta, \theta$ *in radians.* The length s of an arc equal to radius r of a circle is an angle of one radian (57.3º).

Example: Length of arc of 18.2 cm radius and angle 144º. Set "R" on C scale over 144°, read 2.52 radians at index, times 18.2 cm (swap index). Answer is 45.7 cm.

Area of Sector (of circle): $A = \frac{1}{2}r^2\theta, \theta$ in radians. Set HL to r^2 on A scale. Set center B index under HL. Move HL to any 5 on B scale (r^2 x 0.5). If in radians, set center B index under HL, move HL to radians on B scale. If in degrees, slide B scale 57.3 under HL. Move HL to degree angle on B scale (deg. to rad.). Read area on A scale.

Example: sector θ of 90º, radius 440 ft. HL over 440 on D, set center B index under HL, move HL to 5 on B scale. Slide B scale 57.3 under HL then move HL to 90 on B scale, read 152000 ft^2 on A scale. For decimal placement (.5 x 440^2 x 1.57) 0+6+1-1=6, so 152 becomes 152000.
Example: Radius of 14 with sector of 60º. If no A scale, use $\sqrt{}$ scales. Set HL over 14 on $\sqrt{}$, set C index under HL, move HL to 0.5 (C scale), slide radian value of 60º (1.047) of CI scale (to minimize slide movement) under HL, read 102.6 on D scale at index. For decimal placement (.5 x 14^2 x 1.047) 0+3+1-1=3, 1026 becomes 102.6.

Chords: Circle diameter: $d = \frac{(0.5c)^2+h^2}{h}$, where c = chord, h = height from chord to circle edge, d = diameter, r = radius. *If 2 chords intersect, the product of one chord's parts equals the product of the other chord's parts,* $c1a(c1b) = c2a(c2b)$. Simplifying, $(d-h)h = \left(\frac{1}{2}c\right)^2$ becomes $dh - h^2 = \left(\frac{1}{2}c\right)^2$.



Example: Find diameter (d) of inaccessible circle using chord, c = 100 ft, h = 24.7 ft. $d = \frac{50^2+24.7^2}{24.7} = \frac{2500+610}{24.7} = 126ft.$

Example: Find height (h) given diameter of 412, chord of 316. Solving $h = \frac{d-\sqrt{4r^2-c^2}}{2} = 73.8.$ [Quadratic formula.]

Example: Find height given diameter of 126, chord of 100. Set $h = \frac{126-\sqrt{4\times63^2-100^2}}{2} = 24.67.$

For chord angle(θ): $\frac{c}{d} = sin\frac{\theta}{2}$ becomes $\theta = 2\sin^{-1}\left(\frac{c}{d}\right)$.

Example: Radius 63, chord 100, find θ: $\sin^{-1}\left(\frac{50}{63}\right) \times 2 \approx 105º$.

Square Roots, Squares.

$\sqrt{x}, x^2$. Set HL over any number N on the A scale (odd # of digits-left side, even # of digits-right side) and read the square root of N under the HL on the D scale. *See below chart for section. Group in twos. Number of groups (digits or zeros) determine number of digits/zeros in answer.* To find the square of a number, HL on D scale, read A scale.

Slide rules with dual $\sqrt{}$ scales: Set HL to number on D scale. **For roots**, read upper $\sqrt{}$ scale for odd number of digits. Read lower $\sqrt{}$ scale for even number. **For squares**, set number on $\sqrt{}$ scale, odd numbers on lower scale, even numbers on upper scale. Read square on D scale.

Cube Root and Cube.

$\sqrt[3]{x}, x^3$. Set the HL over any number N on K scale and read the cube root of N under the HL on the D scale. One digit, left end; two digits, middle; three digits, right end. Group in threes. Groups to left of decimal determine digits in answer. Reverse for cubes (Set N on D scale, read K scale). *See below chart for scale section to read.*

Another method: Combine squaring and multiplication. Set C index on value on D scale, HL over same value on B scale, read A scale for answer.

Example: 9^3. Index on 9 (D scale), HL on 9 (B), read 729 (A).

(Note) For multiple cube scales, set number on $\sqrt[3]{}$ scale, find cube on D scale. For roots, set on D scale, read on $\sqrt[3]{}$ scales.

Squares, Cubes, and Roots Chart.

For $\sqrt{}$ scales, roots of odd # of digits on upper scale, even on lower scale.

SCALE	SIDE	ZEROS				
D scale		4	3	2	1	0
Sq. A scale ($\sqrt{}$)	L(U) R(L)	9 – 8	7 – 6	5 – 4	3 – 2	1 – 0
Cube, K scale	L M R	14–13–12	11–10–9	8–7–6	5–4–3	2–1–0

SCALE	SIDE	DIGITS				
D scale		1	2	3	4	5
Sq. A scale ($\sqrt{}$)	L(U) R(L)	1 – 2	3 – 4	5 – 6	7 – 8	9 – 10
Cube, K scale	L M R	1–2–3	4–5–6	7–8–9	10–11–12	13–14–15

Quartics, Quintics, Sextics

For slide rules without LL scales, use C, D, A, B and K scales as below.

Quartics $x^4 = (x \times x)^2$ Example: 4^4. Set as normal for multiplication (x^2), but look under HL on A scale, read 256.

Quintics $x^5 = (x \times x)^2 \times x$. Example: 3.7^5. Set as normal for multiplication using the reciprocal (D and CI scales), look under HL as above (A scale, x^4), but slide HL to 3.7 on B scale, read 693 under HL on A scale.

Sextics $x^6 = (x \times x)^3$. For sextics, the K scale is also used. Example: 4.7^6. Set for D and CI scale multiplication (x^2), move cursor to index, read K scale for 10,800.

Logarithms (base 10).

$\log_{10} N$, (log 0.1 = -1, log 1 = 0, log 10 = 1)
If the L scale is on the slide, set the HL over N on the C scale. Read the mantissa under the HL on the L scale. **If the L scale is on the body**, set the HL over N on the D scale. Read the mantissa under the HL (L scale).
(Note #1) If N is greater than or equal to 1, the characteristic is one less than the number of digits to the left of the decimal.

Example: log $\underline{48}$ = $\underline{1}$.681. Example: log $\underline{2540}$ = $\underline{3}$.405

(Note #2) If N is less than 1, set on the CI or DI scale. The characteristic is negative and denoted by an $\overline{\textit{overline}}$ (Modern method uses "-", i.e., -3.5). Its numerical value is the same as the number of zeros between the decimal and the first significant figure in N. **Mantissa is always positive**.

Examples: $\log 0.995 = \bar{0}.0022, \log 0.\underline{00}25 = \bar{2}.602, \log 0.\underline{000}15 = \bar{3}.824, \log 0.\underline{00000}38 = \bar{5}.420$.

Inverse logs: $\log^{-1} x = 10^x$. Set mantissa on the L scale, read number on C (or D) scale. Total digits one more than characteristic.

Example: $\log^{-1}$ 3.497. Read 3.14 (C or D scale). Characteristic is 3 (+1) = 4 digits = 3140.
Example: $\log^{-1}$ $\bar{1}$.375 = 0.$\underline{0}$422 (CI or DI scale, zeros as char.).
Example: $10^{1.45}$: 0.45 (L scale) reads 2.82 (D scale). Characteristic is 1(+1), so 2.82 = 28.2.
Example: Find dB gain if P_{OUT} = 12, P_{IN} = 0.032. $dB = 10 \log \frac{P_O}{P_I}$. Set index at 12 (D scale), HL to 32 (CI scale), read 375 (C scale). Depending on L scale location, align scales and read L on slide, or read L directly on body, for 2.574 x 10 = 25.74 dB. (Multiplying by reciprocal on CI scale extends slide less than dividing on C scale.)

N4 slide rules: Set N on LL scales, read $\log_{10}$ N on D scale, using decimal placement of LL scale in answer. For N < 1, use -LL scales. For **inverse logs**, set on D scale, read LL/-LL scale, using decimal placement of that scale.

Examples: log 4.3 (LL3) = 0.63; log 23 (LL4) = 1.36; log 0.57 (-LL3) = -0.244; log 5^{-3} (-LL4) = -2.3; log 5^{-10} (-LL4) = -9.3.

Natural Logs (base e).

Ln scale: $\ln_e x$. For slide rules that have a Ln scale, set x on D scale (1 < x < 10), read Ln scale direct.

Example: Ln 4.5 = 1.504

Off scale: Find Ln of x in standard form, multiply exponent by Ln 10 (2.3025), add to base value.
Example: Ln 2000 as $2_{10}{}^3$. ln 2 = 0.69. 2.3 x 3 = 6.9 + 0.69 = 7.6.
Example: Ln 481 as $4.81_{10}{}^2$. ln 4.81 = 1.57 + (2.3x2) = 4.6+1.57 = 6.17.

For e^x: set x on Ln scale, read value on D scale.
Example: $e^{1.45}$. Set 1.45 on Ln scale, read 4.26 on D scale.

LL Scales.

Set ln 1.001 to 22026 on LL scales (0.000045 to 0.999 on -LL scales), and read *ln* on D scale, value and decimal according to range (and sign) of LL scale. (On N4 rule, answer is log10. *Ln is read on DF/M scale.*) Decimal placement for reading D scale:

LL0: 0.001 to 0.01, LL1: 0.01 to 0.1, LL2: 0.1 to 1, LL3: 1 to 10.
-LL0: -0.001 to -0.01, -LL1: -0.01 to -0.1, -LL2: -0.1 to -1, -LL3: -1 to -10.

Example: Ln 35 (LL3), read 3.555 (D scale). Example: Ln 1.2 (LL2), read 0.1823 (D scale). Example: Ln 1.03 (LL1), read 0.02958 (D scale). Example: Ln 0.95 (-LL1), read -0.0513 (D scale). Example: Ln 0.0001 (-LL3), read -9.21 (D scale).

N4 scales: Set ln 1.0023 to 10^{10}, read $DF/_M$ scale. Decimal placement:

LL1: 0.001 to 0.01, LL2: 0.01 to 0.1, LL3: 0.1 to 1, LL4: 1 to 100. -LL1: -0.001 to -0.01, -LL2: -0.01 to -0.1, -LL3: -0.1 to -1, -LL3: -1 to -10.LL4: -1 to -100.

For e^x: set x on D scale and read LL scale. If x negative, read -LL scales.

Example: $e^{5.3}$. Set 5.3 on D scale, read LL3 (1-10) for 200.
Example: $e^{0.025}$. Set 2.5 on D, read LL1 (0.01-0.1) for 1.0253.
Example: $e^{-0.056}$. Set 5.6 on D, read 0.9455 on -LL1.
Example: $e^{-4.51}$ = 0.011 on -LL3.
Example: $e^{-.455}$ Set HL to 4.55 on C scale. Slide to right, but value is 1/10 so go down one scale and read .634 (-LL2 scale).

Off scale e^x: Take e^{10} times exponent remainder.

Example: $e^{17} = e^{10} \text{ x } e^7$ = 22026 x 1097 = $24.155_{x10}{}^6$.

Powers of 10: (N>1): Move hairline (HL) to number on LL scale, look at next higher LL scale. Example: 1.35^{10} on LL2 reads 20.1 on LL3. Example: 1.0025^{10} on LL0 reads 1.0253 on LL1.

Powers of 10: (N<1): Uses -LL scales. Move hairline (HL) to number on -LL scale, look at next higher -LL scale. Next higher means next numerically higher (i.e.: -LL01 to -LL02).
Example: 0.9610 on -LL1 reads 0.6648 on -LL2.

Arbitrary Powers: b^x: [Rule of exponents $(A^B)^C = A^{B \times C}$] Set index to *b* on appropriate LL scale, HL to exponent x on C scale. Read answer on appropriate LL scale. (If HL to right, read same scale, if to left, read next higher scale.) Even powers of negative numbers are positive, odd powers are negative.

Example: $9.1^{2.3}$ on LL3. Index to 9.1, HL to 2.3 on C scale. Read 160.6 on LL3.
Example: $1.5^{4.8}$ on LL2. HL to left, so read 7.0 on LL3.
Example: $12^{0.34}$ on LL3. Set index to 12 on LL3, HL to 3.4 on C scale. HL is to right but value is 1/10 so go down one scale and read 2.33 on LL2 scale.
Example: 0.99^{560}. as $(0.99^{5.6})^{100}$. Set index to 0.99 on -LL1 scale, HL to 5.6 on C scale to read 0.945 on -LL1 scale. Raising this to power of 100, look two scales higher to -LL3 scale for 0.0036.
Example: $-25^{1.5}$. Set index to 25 on LL3 scale, HL to 1.5. Read -125 on LL3 scale.
Example: Compound interest rate: 0.015^{12} 1.5% over 12 years. Set left index to 1.015 (decimal percent 0.015) on LL1, HL to 1.2 (12 years) on C scale. Read LL2 scale (>10) for 1.196 (19.6%).

Negative exponents: b^{-x}: Same as above except answer is on -LL scales. If b<1, read reciprocal of answer. Example: $13^{-1.5}$. Set index to 13 on LL3, HL to 1.5 on C scale. Read 0.02133 on -LL3.

Example: 1.5^{-13}. Index to 1.5 on LL2. HL to 1.3 on C scale. Read 0.59 on -LL2, but multiply by 10. So read 0.00514 on -LL3.
Example: 0.985^{-12} (one scale higher). Set index on 0.985 (-LL1), HL on 12, read 0.834 (-LL2) Exponent is negative, so read reciprocal = 1.199.

Finding exponent: b^x=c. Set index over b value on LL scale, HL over c value on appropriate LL scale. Read x value on C scale. *(Exponent's decimal point shifts one place right for each scale higher (b^{10x}); and one place left for each lower ($b^{x/10}$).)*

Example: $6^x = 1.551$. Index over 6 (LL3), HL over 1.551 (LL2), read x = 0.245 (C scale).
Example: $35e^{-.22t} = 0.106$. Isolate $e^{-.22t}$ by dividing 0.106 by 35 = .00303. So $e^x = 0.00303$. Set index over e (LL3), HL over 0.00303 (-LL3), read -5.8. Divide -5.8 by -.22 to obtain t = 26.4.

Finding b given exponent x (roots): $b^x = c,\ or\ \sqrt[x]{c} = b$. Reverse above procedure. Set HL on c value on appropriate LL scale. Slide x value on C scale under HL. Read b value at index on appropriate LL scale. (If index to left, read same LL scale; if index to right, read next lower LL scale.)

Example: $15(x^{0.075}) = 27$. Divide 27/15=1.8 to isolate ($x^{0.075}$). Set HL to 1.8 (LL2), slide .075 (C scale) under HL. Index on right, so one scale lower (LL1), but remember decimal; so go back up two scales to LL3 scale. B = 2533.
Example: $x^{7.5} = 9.5$. Set HL to 9.5 (LL3), slide 7.5 under HL. Index to right, so read next scale lower (LL2), x = 1.35.
Example: $x^{-3.5} = 42$. Set HL to 42 (LL3), slide 3.5 (C scale) under HL. Index is to left but exponent is negative, so read same scale, but reciprocal. x = 0.3437.
Example: Radical $\sqrt[5]{0.138} = b$. Set HL to .138 (-LL3), slide to 5. Index is to right so read next lower scale (-LL2) for b = 0.673.

Off scale/large exponents: Set N on appropriate LL scale. Break exponent into sections such as two or three, that fit on C scale. Square or cube LL scale result. If power is odd or can't fit evenly on C scale, break up as it will fit, such as x^8 set up as $x^5 \times x^3$.

Example: 8^9. Set up as $(8^{4.5})^2 = 11585^2 = 1.342_{10}{}^{8}$.
Example: 5^{11}. Set as $(5^{5.5})^2 = 6988^2 = 4.88_{10}{}^{7}$.
Example: 15^9. Set as $(15^3)^3 = 3375^3 = 3.84_{10}{}^{10}$.
Example: -25^8. Set as $(-25)^5$ x $(25)^3 = -1.526_{10}{}^{11}$.

Numbers close to 1: Imagine smaller scales, and add zeros (>1) or nines (<1) as required.

Example: $1.03^{0.02}$. 0.02 is two scales lower. Set index at 1.03 (LL1), HL to 2 (C scale). Imagine a lower scale below LL0 (LL0-1). Read 1.00594 (LL0), add a zero for 1.000594.
Example: $0.978^{0.0023}$. 0.0023 is 3 scales lower. Set index at 0.978 (LL1), HL at 23 (C scale). Imagine a scale ($-LL0_{-2}$). Read 0.9949 (-LL0), add two nines for 0.999949.

Roots: Set HL to value on LL scale, slide root value under HL, read root at index (*If index to right of value, use next lower LL scale, if index to left, use same scale. For each exponent zero, use next lower scale. For decimal values, if index to right, use same scale, if index to left, use next lower scale. For each decimal zero, use next lower scale.*).

Example: $5^{0.1}$ (or radical) $\sqrt[10]{5}$ on LL3. Read 1.175 on LL2.
Example: $\sqrt{4.95 \times 10^{-16}}$. Set HL to 4.94 on LL3, slide 2 on C scale to HL, read $2.22_{10}{}^{-8}$ on LL2 under right index.
Example: $\sqrt[3]{729} = 9$. Set HL to 729 (LL4), slide 3 (C scale) under HL, read 9 (LL3) at right index.
Example: $\sqrt[7]{150} = 2.045$. Set HL to 150 (LL4 scale), slide 7 (C scale) under HL, read 2.045 (LL3

scale) at right index.
Example: $\sqrt[0.03]{0.964}$. Set HL at .964 (left side -LL2 scale), set 0.03 under HL, read 0.295 (-LL3 scale) at right index.
Example: $\sqrt[0.03]{0.85} = 0.0044$. Set HL at .85 (right side -LL2 scale), set 0.03 under HL, read 0.0044 (-LL4 scale) at left index.

Logs to any base: $\log_b N = x$, see as $b^x = N$ (see 'Finding exponent' above).

Example: $\log_2 5 = x$ is $2^x = 5$. Index on 2 (LL2), HL over 5 (LL3), read 2.322 (C scale).

No LL0 scale: The D scale can be used as a LL0 scale. Consider the left end as 1.001 and the right as 1.01. Error $\approx$ 0.2% at right end and negligible at left end.

Example: ln 1.003 = 0.0030. Error 1.7% (0.002995).
Example: 1.0025^3. Set index on 25 of D scale, HL to 3 on C scale. Read 1.0075 on D scale (error $<$ 0.002%).

No LL scales: N^{exp} Write $10^x = N$. Set N on D (or C) scale to find log N = x. Then $(10^x)^{exp}$. Multiply exponents. Set result on L scale. If result negative, set on L scale Right to Left. Read answer on D (or C). *Use standard notation to locate decimal.*

Example: $1.5^{2.4}$. Write $10^x = 1.5$. Set 1.5 on D (or C), find x = 0.176 on Log scale. Then $(10^{0.176})^{2.4} = 10^{0.422}$. Set 0.4224 on L, read 2.646 on D.
Example: $13^{-1.5}$. Write $10^x = 13$. Set 13 on D (or C) scale. Find log 13 = 1.114. So $(10^{1.114})^{-1.5}$ = -1.671. Set -1.671 on log (R to L) and read 2133. Characteristic of -1 is one zero, so answer is 0.02133.

Natural log to common log: $\log_{10} x = \frac{\ln x}{2.3026}$; or, $\ln x \times 0.4343$.

Example: $\log_{10} 35$. $\frac{\ln 35}{2.3026} = \frac{3.555}{2.3026} = 1.544$. On the N4 with LL and $DF/_M$ scales, read the common log directly. Set HL on 35 on LL3 scale, read $DF/_M$ scale value of 1.544. On others, to convert, set the same, but read 3.555 on the D scale, then set C scale's 2.3026 under HL (divide), read 1.544 at index on D scale. Or, more directly, set index to "10" on LL scale, slide HL to x value on LL scale, read answer on C scale.
Example: $\log_{10} 0.55$. $\ln 0.55 = -0.598 \times 0.4343 = -0.26$. Set HL at 0.55 on the -LL2 scale, read .598 on D scale, set index there, move HL to 0.4343 on C scale (multiply), read -0.26 on D scale. Or again, set index to "10" on LL scale, slide HL to 0.55 on -LL scale, read C scale. Verify by setting 0.55 on CI (or DI) scale, read **L** scale for mantissa.

Exponents log format: $\log a^b = b \times \log a$.

Example: $\log_{10} 5^3 = 3 \times 0.699 = 2.097$.
Example: $\log_{10} 5^{-3} = -3 \times 0.699 = -2.097$.
Example: $\ln 5^3 = 3 \times 1.61 = 4.828$.
Example: $\ln 5^{-3} = -3 \times 1.61 = -4.828$.

Sine/Cosecant; Cosine/Secant of an Angle.

$\sin x, \csc x; \cos x, secx$. (Values: C scale 0.1 to 1.0, CI scale 1 to 10.)

Sine, cosecant: HL over x on the S scale (L→R) and read:

- sin x on the C scale, cosecant x on CI scale .

Cosine, secant: HL over complement of x on S scale (L←R) and read:

- cos x on C scale, secant x on CI scale.

Examples: sin 44 = 0.695, csc 48 = 1.346; cos 44 = 0.719, sec 35 = 1.22.

Tangent/Cotangent of angle.

$\tan x, \cot x$ (Values: C scale 0.1 to 1.0, CI scale 1 to 10.) Set the HL over x on the T scale and read:

- Angles < 45º: tan x on the C scale, cot x on CI scale.
- Angles > 45º: tan x on the CI scale, cot x on C scale.

(Note) For inverse values, set the HL on the value (C or CI scale) and read the angle on the T scale (i.e. arc tan ($\tan^{-1}$ x), arc cot ($\cot^{-1}$ x)).

Dual-T scales: Read tan/arc tan on C scale only, cot/arc cot on CI scale only.

- tan < 45º, value < 1; tan > 45º: value >1.
- cot < 45º, value > 1; cot > 45º, value < 1.

Examples: tan 35º = 0.700, tan 50º = 1.19.
Examples: tan-1 2.75 = 70º, tan-1 0.268 = 15º.
Examples: cot 35º = 1.428, cot 50º = 0.84, cot 82º = 0.141.
Examples: $\cot^{-1}$ 3.73 = 15º, $\cot^{-1}$ 2 = 26.55º.

Trig Functions, Angles < 5.7 degrees.

(Values: C scale 0.01 to 0.1, CI scale 10 to 100.) Set the HL over the angle on the ST scale. Read the sine and tangent under the HL on the C scale. Read the cotangent on the CI scale. For cosines, use *cos x = sin(90 - x)*.

The sine and tangent of angles less than 5.7º are very nearly equal and are between 0.01 and 0.1.

Examples: sin 2.3º = 0.04013, tan 2.3º = .04016.
Example: cos 0.62º. 90º-0.62º=89.38º. Sin 89.38º = 0.9999.

Angles < 0.57º: Read ST scale as if the decimal point were at left of numbers printed, and read C scale with decimal point one place to left of normal. Range is <0.001 to ~0.01.

Examples: sin 0.2º = 0.00349; tan 0.16º = 0.00279.
Examples: cos 0.25. 90º-0.25 = 89.75º. Sin 89.75º =0.99999.

Angles > 84.3º: *For cosines > 84.3º*, obtain complement (90º-angle), set on ST, read value on C scale (range 0.01 to 0.1).

Example: 32 cos 85.2º: (90º-85.2º = 4.8º). Index to 32 (C scale), HL to 4.8º (ST scale), read 2.678 (C scale). Cos 85.2º = 0.0837.

For tangents > 84.3º, subtract from 90º, set on ST, read cotangent (CI scale) ($\frac{1}{\tan x}$). Range 10.0 to ∞, so $\lim_{x \to 90} \tan x = \infty$.

ST scale can also be read backwards where 5° = 85° and 1° = 89° (match whole degrees). Results (10-100) on CI scale.

Example: tan 88.59° = (90°-88.59°) = 1.41. Set 1.41 on ST scale. Read cotangent of 1.41 = 40.63 on CI scale.
Example: 520 tan 89°: HL on 520 (D scale), slide 1° (ST scale) under HL, read 29,790 on D scale at index. Tan 89° = 57.29 ($360 \div 2\pi$).

Angles > 90°; negative angles: Reference angle is acute angle from the x axis. Negative angles are clockwise from x axis.

Example: sin 217°. 217°-180°=37°. Given angle is positive and in the third quadrant, so sine is negative: -sin 37°= -0.602.
Example: tan -114°. 180°-114° = 66°. Given angle is negative and in third quadrant, so tangent is positive: tan 66° = 2.25.
Example: cos 126°. 180°-126°=54°. Given angle is positive and in second quadrant, cosine is negative: -cos 54° = -0.588.

Quadrant			**I**			**II**	**III**	**IV**
Reference			θ			**180 -** θ	θ **- 180**	**360 -** θ
Angle	0°	30°	45°	60°	90°	180°	270°	360°
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
Sine	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	+ 1 +	+ 0 -	-1	- 0
Cosine	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	+ 0 -	- 1 -	- 0 +	+ 1
Tangent	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	∞	- 0 +	∞	- 0
Cotangent	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	+ 0 -	∞	+ 0 -	∞

To determine answer sign, note which quadrant the angle falls.

No ST scale: For small angles, the sine or tangent functions can be approximated closely by the equation: $\sin x = \tan x = x/(180/\pi) = x/57.3 = radian(x)$. Given this, it becomes a simple division. Set the C index to 57.3 on D scale. Most D scales have an 'R' there. Set the HL to the angle on D scale. Read the sine or tangent on the C scale.

Example: sin 2.5°. Set as above, with HL on 2.5 (D scale), read 0.044 (C scale).

Hyperbolic Scales (SH/TH), N4 slide rule.

$\sinh x = \frac{e^x - e^{-x}}{2}$; $\cosh x = \frac{e^x + e^{-x}}{2}$; $\tanh x = \frac{\sinh x}{\cosh x}$; $\coth x = \frac{\cosh x}{\sinh x}$; $sech\ x = \frac{1}{\cosh x}$; $csch\ x = \frac{1}{\sinh x}$.

Sinh x: Set HL over x (SH scale), read sinh x (C scale). Upper SH scale, decimal left of number, $0.1 \leq \sinh x \leq 1$; lower SH scale, decimal to right of first digit, $1.0 \leq \sinh x \leq 10$.

Examples: sinh 0.5 = 0.521; sinh 1.3 = 1.7; csch 0.5 = 1.92.

Cosh x: $\cosh x = \frac{\sinh x}{\tanh x}$. Align C and D scales. Set HL over x on SH scale, move slide to place x on TH scale under HL. Read cosh x on D scale at index.

Alternate method using LL scales. Set x on DF/$_\mathrm{M}$ scale, read e^x and e^{-x}, add and divide sum by 2. If $x<1$ read next lower LL/-LL scales. For rules with no DF/$_\mathrm{M}$ scale, set x on D scale.

Example: cosh 0.54 = 1.15.; sech 0.54 = 0.87.
Example: cosh 0.434. Use alternate method. Set 0.434 on DF/$_\mathrm{M}$ scale, read e^x on LL3 and e^{-x} on -LL3, add and divide sum by 2 for 1.095.
Example: For rule with no DF/$_\mathrm{M}$ scale. For cosh 1.85, set 1.85 on D scale, read LL3/-LL3 (1.0 - 10.0) scales for 6.35 and 0.158, divide sum by 2 for 3.26.

Tanh x: $\tanh x = \frac{e^{2x}-1}{e^{2x}+1}$. Set HL over x on Th scale, read tanh x on C scale. Decimal is to left of number ($0.1 \le \tanh x \le 1$). Larger values approach 1.00.

Examples: $\tanh 0.4 = 0.38$; $\tanh 0.176 = 0.174$.

Large values: For x < 3, both $\sinh x$ and $\cosh x$ are $\approx e^x/2$. Accuracy for x >10 is poor.

Rules with DF/$_M$ scales: Set x on DF/$_\mathrm{M}$ scale, read +LL4 scale and mentally divide by 2.

Example: sinh 12 or cosh 12 = 81377.

Rules without DF/$_M$ scales: Set x on D scale, read +LL3 scale and mentally divide by 2. $1 \le x \le 10$.

Example: sinh 4 or cosh 4 = 27.3.

Small values: For x < 0.10 sinh and tanh are ≈ x, and the cosh x ≈ 1.

DF/$_\mathrm{M}$, CF/$_\mathrm{M}$ Scales.

The DF/$_\mathrm{M}$, CF/$_\mathrm{M}$ scales are folded at 1/M=2.3026 where modulus M=$\log_{10}$e (0.4343). Shows directly natural log of number from LL scales. Where most rules show answer on D or C scales, the N4 slide rule shows answer here. See 'LL Scales' section for decimal placement in answer (*If answer is left of 1 on DF/$_M$ scale, move decimal left one place*). Some slide rules have answer range printed at end of LL scales. These scales can also be used as normal DF/CF scales.

Examples: ln 2.2 (+LL3) = 0.788 (DF/$_\mathrm{M}$); $e^{3.4}$ (DF/$_\mathrm{M}$) = ln 30 (+LL4); $e^{13.4}$ (DF/$_\mathrm{M}$) = ln 660k (+LL4); ln 0.05 = -2.996.

Special Graduations.

0.7854 = $\pi/4$ near right-hand end of A][B,C][D, near center on CF][DF scales. Use for area calculations.

R on DI, D][C,CI, CIF and CF][DF scales, marks 57.3, to convert radians to degrees and trig functions. See Radians to Degrees section above.

' (minute mark) at about 1.97$^\mathrm{o}$ on the ST scale. When this is set opposite a minute value on D scale, sine/tangent is shown at C index on D scale, *as is radians at the same setting.*

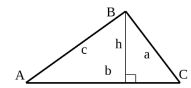
Example: sin 1' = 0.00029. "*Three zeros and a three.*" Each minute increases by the same amount.
Example: tan 6' = 0.00175.

“ (second mark) at about 1.18º on the ST scale. It is used the same as the minute mark.

Example: sin 1” = 0.00000485. “*Five zeros and a five.*” Each second increases by the same amount. Example: sin 30” = 0.000145.

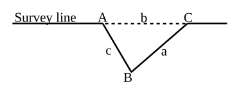
Triangles, Law of Sines.

$\frac{a}{\sin a} = \frac{b}{\sin b} = \frac{c}{\sin c}$. C scale over D scale: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



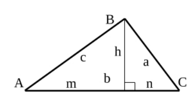
Oblique triangles: Aº + Bº + Cº = 180º, h = c sin A = a b a sin C, Area = 1/2 base height. To use the law of sines, we must have a known side opposite a known angle. If the side opposite the given angle is smaller than the other given side, there may be two possible solutions as supplementary angles have same sine values. So find 2nd angle and subtract from 180°. Check validity by adding this angle to given angle. Valid if total < 180°. For angles > 90: $\sin(180 - \theta)$. If values are off scale, reverse C scale end-for-end (2nd example).

Example: A = 35º, a=42, B=70º, C = 180º – (A + B) = 75º. $\frac{\sin 35}{42} = \frac{\sin 70}{b} = \frac{\sin 75}{c}$ Set 35º on S opposite 42 on D. Move HL over 70º on S, read b = 68.8 on D. Move HL over 75º on S, read c = 70.7 on D. h = 68.8 sin 35 = 39.5.



Surveying: Determine inaccessible distance A to C for survey work. Line AB distance is measured and transit angles A and B are measured. Then C = 180º - A - B. So then the A to C distance: b = sin B(sin C/c). And B to C distance: a = sin A/(sin C/c).

Example: A = 73.3º, c = 54 ft., B = 101º. C = 180º - 73.3 - 101 = 5.7º. Then the A to C distance, b = sin 101/(sin 5.7/54) = 533.7 ft., and the B to C distance, a = sin 73.3/(sin5.7/54 = 520.8 ft. Example: A=55.3°, a=22.8, b=24.9. Set 55.3° over 22.8, read 63.9° over 24.9. 180-(A+B)=60.8 which gives c=24.2. As b>a, is ambiguous case. Subtract 180-63.9=116.1° supplementary angle. As 116.1 + 55.3 < 180°, this is valid. Remainder is 8.6°. Set HL to 8.6°, read 4.14 on D scale. Two triangles shown as: $\frac{55.3}{22.8} = \frac{63.9}{24.9} = \frac{60.8}{24.2} \Leftrightarrow \frac{55.3}{22.8} = \frac{116.1}{24.9} = \frac{8.6}{4.15}$



Oblique: 2 sides, but unknown side a opposite the angle. Ref: $\frac{\sin A}{\frac{1}{c}} = \frac{\tan A}{\frac{1}{m}} = \frac{\tan C}{\frac{1}{n}} = \frac{\sin C}{\frac{1}{a}} = h$. Determine h: c sin A, then find m: $\frac{h}{\tan A}$. Find n: b=m=n. Find C: $\tan^{-1}\frac{h}{n}$. Find a: $\frac{h}{\sin C}$, then B: 180-(A+C).

Right triangles: Read S and T scales (tan > 45 and cot < 45, read CI scale): $\sin\theta = \frac{opposite}{hypotenuse}$; $\tan\theta = \frac{opposite}{adjacent}$; $\cot\theta = \frac{adjacent}{opposite}$; $\cos\theta = \frac{adjacent}{hypotensue}$. a = opposite, c = hypotensue, b = adjacent.
$b = c\cos A = a\cot A = c\cot A = c\sin B = a\tan B = \sqrt{(c+a)(c-a)}$.
$a = c\sin A = b\tan A = c\cos B = b\cot B = \sqrt{(c+b)(c-b)}$.

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A \rightarrow \cos A = \frac{b^2+c^2-a^2}{2\times b\times c}$; $b^2 = a^2 + c^2 - 2ac\cos B \rightarrow \cos B = \frac{a^2+c^2-b^2}{2\times a\times c}$; $c^2 = a^2 + b^2 - 2ab\cos C \rightarrow \cos C = \frac{a^2+b^2-c^2}{2\times a\times b}$.

Example: a = 10, b = 12, c = 14. $\frac{\sin A}{10} = \frac{\sin B}{12} = \frac{\sin C}{14}$. Setup the slide rule to determine the A angle shown: $\cos A = \frac{12^2+14^2-10^2}{2\times 12\times 14} = \frac{240}{336} = 0.714$. Set HL to 7.14 on C scale. Read the angle 44.4º on S scale for cosines (right to left). Then determine the remaining angles using the law of sines: $\frac{\sin 44.4}{10} = \frac{\sin 57}{12} = \frac{\sin 78}{14}$. Align indexes, move HL to 44.4 on S scale (left to right) shows 0.07 on C scale, then multiply by 12 (HL to 12), line up indexes to read 57º under HL on S scale. Place

HL back to 0.07 on C scale, multiply by 14 (HL to 14), line up indexes to read 78º under HL on S scale.

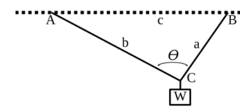
Obtuse angles: The cosine of angles >90º to 180º is negative, so change minus sign in above three cosine formulas to plus (i.e., a2 = b2 + c2 + 2bc cos A). Then use $\cos(180-\theta)$ in formulas.

Example: B = 126º, a = 13, c = 22. Find cos(180º-126º) = cos 54º. $b = \sqrt{13^2 + 22^2 + 2(13)(22)\cos 54} = \sqrt{169 + 484 + 336} = \sqrt{989} = 31.45$. Law of sines gives $\frac{\sin 19.5}{13} = \frac{\sin 126}{31.45} = \frac{\sin 34.5}{22}$.

Trig Reciprocals: $\sin x = \frac{1}{\csc x}$; $\csc x = \frac{1}{\sin x}$; $\cos x = \frac{1}{\sec x}$; $\sec x = \frac{1}{\cos x}$; $\tan x = \frac{1}{\cot x}$; $\cot x = \frac{1}{\tan x}$.

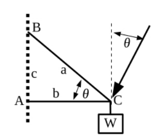
Trig Complements: $\cos x = \sin(90-x)$; $\cot x = \tan(90-x)$; $\csc x = \sec(90-x)$.

Vectors: Determining equilibrium or tensions on cables/structures: $b = \frac{W}{\sin\theta}\cos B$, $a = \frac{W}{\sin\theta}\cos A$.



Example: 800 lb. suspended with two cables. A = 30º, B = 45º. $b = \frac{800}{\sin 75}\cos 45$, $a = \frac{800}{\sin 75}\cos 30$. Tension on each? As $\theta = 105$, 180º-105º=75º, set HL to 800 (D scale), slide sin 75º (S scale) under HL. Move HL to cos 45º, read b=586 (D scale). Move HL to cos 30º, read a=717 (D scale).

Example (law of sines): Suspended 100 lb weight from two bars, angle at joint is 60°. Tension and force? $a = \frac{W}{\sin\theta}$, $b = a\cos\theta$. Align sin 60° with 10 (D scale), read at left index (90º), tension on a = 115.5. Move HL to cos 60, read force on b = 57.7. If W force is angled, use $W\cos\theta$ to determine actual weight W to use.



Height of Object: Determining height of object given angle to top and distance from base. $\tan\theta = \frac{h}{d}$ becomes $h = \tan\theta d$.

Example: 64° angle from 25 feet away. Set index at 25, slide HL to $\tan 64$ (2.05), read h=51.25 (D scale). Determine hypotenuse by Pythagorean theorem or $m = \frac{d}{\cos\theta}$. Set $\cos 64$ (0.438) over 25 (D scale), read m=57 at index.

Additional Information

General Information.

[DSP, FFT][6] Cardinal sine function. $\lim_{x\to 0} sinc\ x = 1$.

- Mathematics: $sincx = \frac{\sin x}{x}$ (non-normalized),
- Digital signal processing: $sincx = \frac{\sin \pi x}{\pi x}$ (normalized).

[6]The ARRL Handbook for Radio Communications, 2016, Chapter 15.

Convert feet, inches to decimal feet: ft + (in 8.33 /100)
Dº M' S" to Decimal Degrees: Dº + (M' / 60) + (S" /3600)
1 nautical mile = 6076 ft., 1.151 statute mile, 1852 meters.
Acceleration, gravity: 32.174 f/s^2, 9.807 m/s^2
Sea level pressure: 14.7 lb/in^2, 1.013x105 Pa.
Degrees C to F: 1.8 x C + 32, F to C: .555 (F – 32)
Absolute zero = -273.15ºC = -459.67ºF = 0º Kelvin
Speed of sound: 1125.33 f/s, 343 m/s
E=MC2 (Energy=Mass C2), C=300M m/sec. = 186K mi./sec.
π = 3.141592653589793238462, e = 2.718281828, 1 radian = 57.29577951º
2 radians = 6.283185307 radians = 360º
Circle circumference = $\pi \times diameter$; circle area = πr^2
Volume cylinder = $\pi r^2 h$ = 0.7854 x diameter^2 x height.
1 acre = 43560 ft^2 = 4046.87 m^2, 1 section = 640 acres.
1 oz = 437.5 grains = 28.35 grams, 1 troy oz = 31.1035 grams.
1 hp = 746 watts = 33,000 foot pounds per minute.
Terminal velocity: $V = P \tanh \frac{2gt}{P}$, $P = \sqrt{W/k}$, W=mass, k=0.0037, t=time, g=32.2 ft/sec^2.
Hydraulics:[7] P1=P2 (Pascal's principle); $\frac{F_1}{A_1} = \frac{F_2}{A_2}$, $F_2 = \frac{F_1 A_2}{A_1}$, $F_1 = \frac{F_2 A_1}{A_2}$. P = pressure, A = area, F = force.
1 liter = 61.02 in^3 = 1000 cm^3
100 mils = 1/10 inch = 2.54 mm. $\frac{1}{2.54} = 0.3937$. 1000 mils = 1 in. = 25.4 mm.

Quadratic Equation.

$ax^2 + bx + c = 0$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ To factor a quadratic equation, set index on c value (D scale). Then, any product of simultaneous CI and D scale reading (or CIF, DF) is equal to c. Then move the HL so the sum of the CI and D scale readings (or CIF, DF) equals b. Check by plugging in values, answer should equal c (opposite sign). If a is present, multiply by c, set index to product. **Important: pay attention to signs**.

Example: x^2 + 8x + 15 = 0. Index to 15 (D scale), move HL until CI and D equals b (roots: -5, -3). Check by inserting into formula: (-5)^2 + (8(-5)) = -15, (-3)^2 + (8(-3)) = -15.
Example: 2x^2 + 8x - 20 = 5x. First, move terms to left-side, 2x^2 + 8x - 20 - 5x = 0, combine like terms, 8x - 5x = 3x, for 2x^2 + 3x -20 = 0. Reducing 2x^2 as 2 x -20 = -40 gives x^2 + 3x -40 = 0. Index on 40, find 5, -8 roots, divide by 2 again to restore, for ending roots of 2.5, -4. Check: 2(2.5)^2 + 3(2.5) = 20, 2(-4)^2 + 3(-4) = 20.
Example: 3x^2 + 7x - 21 = 0. Reducing 3x2 as above gives x^2 + 7x - 63 = 0. Index on 63, find roots -12.174, 5.175. Dividing by 3 gives -4.058, 1.725. Insert values to check: 3(-4.058)^2 + 7(-4.058) = 21, 3(1.725)^2 + 7(1.725) = 21

Electronics.

W = EI = E^2/R = I^{2}R;
I = E/R = W/E = $\sqrt{W/R}$;
R = E/I = E^2/W = W/I^2;

[7]See also Proportion and Percentage section.

E = IR = W/I = $\sqrt{WR}$; (Z $\Leftrightarrow$ R);
$I = \sqrt{\frac{W}{Z\cos\theta}} = \frac{P}{E\cos\theta}$;
$E = \frac{P}{I\cos\theta} = \sqrt{\frac{PZ}{\cos\theta}}$;
$Z = \frac{P}{I^2\cos\theta} = \frac{E^2\cos\theta}{P}$;
$P = I^2 Z\cos\theta = IE\cos\theta = \frac{E^2\cos\theta}{Z}$;
Power factor: $pF = \cos\theta = \frac{P}{EI} = \frac{R}{Z}$;
$dB = 20\log\frac{I_{out}}{I_{in}} = 20\log\frac{E_{out}}{E_{in}} = 10\log\frac{P_{out}}{P_{in}}$;
dB ratio: 1/2 voltage, current = 6 dB; 1/2 power = 3 dB.
Power conversion (referenced to 1 mW [0 dBm = 1 mW]):

- volts to dBm: $dBm = 10\log\frac{E^2/R)}{0.001}$
- dBm to volts: $uV = \sqrt{\log^{-1}(dBm/10)0.001R}$.

Time constant = L/R = RC. Tc = 63, 86, 96, 98, 99%.
Resistance: Series resistances add. Parallel resistances, the total is less than smallest value, but not less than half (useful for decimal placement).
$R_t = \frac{R_1\times R_2}{R_1+R_2} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_n}}$.
Parallel capacitance adds, series capacitance is reciprocal.

Resonant Circuits: $Z = \sqrt{\frac{L}{C}}$. Handy formulas for finding C and L: As $\frac{1}{(2\pi)^2} = 0.02533$, then $L = \frac{0.02533}{f^2C}$ and $C = \frac{0.02533}{f^2L}$. Set HL over 2533 (even # digits, right side) on A scale. Slide frequency on C scale under HL. Place HL over B index. Slide L (or C) value on B scale under HL. Read C (or L) on A scale at index. As a guide for decimal placement, 10 uH and 10 pF give about 16 MHz resonant frequency.

Example: 35 MHz, with 8.4 uH gives C = 2.46pF. For decimal: $\frac{0.02533}{(35^E 10^6)^2\times 8.4^{-6}} = \frac{0.02533}{1225^{12}\times 8.4^{-6}} = \frac{25330^{-6}}{10290^6} = 2.46^{-12}$.

Resonant frequency: $f_r = \frac{1}{2\pi\sqrt{LC}}$; 2π radians = 1 Hz, and as $\frac{1}{2\pi} = 0.159$, then also $f_r = \frac{0.159}{\sqrt{LC}}$.

Example: 8uH, 10pF. $f_r = \frac{0.159}{\sqrt{8\times 10}}$. Multiply 8 x 10 = 80. Slide right side (even digits) of B scale 80 over 1.59 on D scale. Read 1.78 on D scale at index. For decimal: $\sqrt{80^{-18}} = 8.94^{-9}$, $\frac{15.9^{-2}}{8.94^{-9}} = 1.78^7 = 17.8MHz$. For the N4 rule (or Post 1460) which has no A/B scales, set left index to 0.159 on D scale, slide HL to 80 (D scale) to determine square root from $\sqrt{}$ scales on stock, note value. Slide HL to that value on CI scale, read 1.78 on D scale.

Inductive reactance: $X_L = 2\pi fL$. Using DF scale (make π the last operation). Multiply f or L by 2. Set index to one value on D scale, HL to other value on C scale. Read reactance on DF scale.

Example: 5 Henry coil at 30 Hz. Set index at 30, HL at 10 on C scale (index), read 942Ω on DF scale.
Example: 60 mH coil at 1200 Hz. Index at 2400 (D scale), HL at 60 (C scale), read 4.524 (DF scale). Standard notation: $2.4_{10}^{3}\times 6.0_{10}^{-2}\times\pi = 452.4\Omega$.

To determine frequency for a particular XL, set HL on ohms (DF scale), slide twice inductance value (C scale) under HL, read frequency at index on D scale. For L value, slide twice frequency (C scale) under HL, read inductance on D scale at index.

Example: X_L of 600Ω, L of 120 mH. Set HL on 6.00 (DF scale), slide twice L (2.4) on C scale under HL, read 7.95 (795 Hz) at D scale index.
Example: X_L of 450Ω, frequency of 840 kHz. Set HL on 450 (DF scale), slide twice frequency (1680) on C scale under HL, read 8.5 (85 mH) at D scale index.

Capacitive reactance: $X_C = \frac{1}{2\pi fC}$. Also $X_C = \frac{0.159}{fC}$ as $\frac{1}{2\pi} = 0.159$.[8] Do same as above section, but align scales & read reciprocal of DF on CIF scale. Check reciprocal on LL scales if desired.

Example: 60 uF at 1200 Hz. Double frequency and set index to 2.4 on D scale. Move HL to 6 on C scale. Read 45.24 on DF scale. Align scales and read CIF scale for reciprocal of 2.21. Standard notation: $\frac{1}{2.4_{10}{}^{3} \times 6.0_{10}{}^{-5} \times \pi} = 2.21\Omega$.
(Note: ω sometimes used for $2\pi f$, i.e. $X_L = \omega L, X_C = \frac{1}{\omega C}$)

To determine frequency for a particular XC, set HL on ohms (aligned CIF scale), move index to twice capacitance value (D scale), read frequency on C scale under HL. *For C value*, index to twice frequency (D scale), read capacitance on C scale.

Example: For X_C of 15 Ω, C of 15 uF, set HL on 15 (aligned CIF scale), move index to 30 on D scale, read frequency of 7.07 (707 Hz) under HL on C scale.

Ratios:[9] XL is directly proportional to frequency, $\frac{f_1}{f_2} = \frac{X_L 1}{X_L 2}$.
XC is inversely proportional to frequency, $\frac{f_1}{f_2} = \frac{X_C 2}{X_C 1}$.

Example: XL is 5425 @ 50 Hz. What is XL @ 60 Hz? $\frac{50}{60} = \frac{5425}{X_L}$. $X_L = 6510$.
Example: XC is 636 @ 50 Hz. What is XC @ 60 Hz? $\frac{50}{60} = \frac{X_C}{636}$. $X_C = 530$.

Pythagorean Theorem short cut: $a = \sqrt{b^2 + c^2}$; or $z = \sqrt{r^2 + x^2}$; or $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Single-T scale: Set the C scale index (try left first) over smaller number on D scale. Set HL over larger number on D scale. Note number under HL on B scale. If number is on right half of the slide, number is read as a two-digit number. Add 1 to this number (if r,x ratio >10, add 0.01, left side; or 0.1, right side) and move HL to this number on B scale. Read a on D scale. Estimate answer to be larger than greatest number, but less than the sum.
If no A/B scales, use **R** scales instead.[10] Set index to 1st "R" number, slide HL to 2nd "R" number, **add 1** to C scale reading, move HL, read answer on R scale at HL.
Phase angle: $\tan\theta = \frac{X}{R}$ (rectangular to polar coordinates). If X > R set quotient on CI scale, read cotangent on T scale (R to L).

Example: $z = \sqrt{7^2 + 9^2}$. Set index to 7 on D scale, HL to 9 on D scale. Note 1.65 on B scale. Add 1 for 2.65 which is off-scale. Move HL to index, swap ends, and set HL to 2.65 on B scale. Read 11.4 on D scale. For phase angle, divide 9 by 7 for $\tan\theta = 1.285$. As X>R, set on CI scale and read cot 1.285 = 52.1^{O} on T scale ($L \Leftarrow R$). Answer $z = 11.4\angle 52^o$.

Dual-T scales: $Z = \sqrt{r^2 + x^2}$. Set index to resistance (r), HL to reactance (x) (both on D scale), note phase angle (top T scale if r larger, bottom T scale if x larger). Slide sine (S scale) under HL and note impedance (z) at index. (*If θ is small (x/r < 0.1), read on ST scale. If x/r > 10, subtract ST reading from 90^o. If ratio > 100, move ST decimal left one place for better approximation. Error ≈ 1%*).

[8] See also N-515-T section, 2π scale.
[9] See also Proportion and Percentage section.
[10] The Post 1460 has "R" scales, the K&E 68-1100 has "Sq" scales.

Example: 38+j21. Set index on 38 (D scale), HL on 21 (D scale). Read phase angle on upper T scale as 29°. Slide 29 (S scale) under HL, read 43.4 at index. Answer $z = 43.4\angle 29^o$.
Example: 15-j7799. Index to 15, HL to 7799. Ratio ≈ 500, so read ST scale for 2.97, move decimal to 0.297°. 90°-0.297° = 89.7°. Slide S scale 89.7° under HL, read $7799\angle 89.7^o$.

Impedance for Series circuits, Single-T scale only: $Z = \sqrt{R^2 + X^2}$. Simpler impedance, phase angle method (rectangular to polar coordinates transformation). Set C index to larger number on D scale. Set HL to smaller number on D scale, and read θ on T scale. If x < r then $\theta < 45$°, read left to right. If x > r then $\theta > 45$, read right to left. Move slide until θ on S scale is under HL (same direction as T). Read Z on D scale at index. (*If θ is small (x/r < 0.1), read on ST scale. If x/r > 10, subtract ST reading from 90°. If ratio > 100, move ST decimal left one place for better approximation. Error ≈ 1%*)

Example: (6+j7). $Z = 9.22\angle 49.4^o$.
Example: (2+j7). $Z = 7.28\angle 74^o$.
Example: (14.6+j9.4). $Z = 17.36\angle 32.8^o$.
Example: 110VAC, 8.3Ω @ 13.25A. Series reactance needed for same current @ 220 VAC? Find total Z to be 220/13.25=16.6Ω. Set C index at 8.3 (D scale), HL to 16.6 (D scale), read 4 (B scale), subtract 1, set HL to 3. Find needed reactance to be 14.4Ω (D scale). Now, setting index at 14.4, HL at 8.3 gives θ of 60° (T scale, R to L). Power factor = 0.5 or 50%.

1/4 wave transformer to match transmission line to real load. $Z_1 = \sqrt{Z_0 Z_1}$.

Power factor: $pF = \cos\theta$. Angle inversely proportional to percentage.

Example: cos 51.4° = 0.624 = 62.4%.

Power dissipation: $diss. = \frac{E^2 \cos\theta}{Z}$.

Example: Z of 55Ω @ 80vac, phase angle as above. $\frac{80^2 \cos 51.4}{55} = 72.6w$.

Circular mil (CM) area of rectangular conductors: $A = \frac{4ab}{\pi}$, where a and b are dimensions in mils. Example: 1/4 x 1/2 inch, $A = \frac{4 \times 250 \times 500}{\pi} = 159.1k$ cir. mils. As 4x250x500=5000, set HL to 5000 on DF scale, read 1591 on D scale.

Current carrying capacity: ≈ 700 CM per amp. $CM = \frac{Amps \times feet \times 21.6}{drop}$. Drop calculated at 3 % (0.03).

Impedance for Parallel circuits: $Z = \frac{RX}{\sqrt{R^2+X^2}}$, Phase angle = 90° - angle° = $\tan^{-1}\frac{I_X}{I_R} = \tan^{-1}\frac{R}{X}$, $I_T = \sqrt{{I_R}^2 + {I_X}^2}$.

Example: (300+j400). RX = 120,000, $\sqrt{R^2 + X^2} = 500$. Angle 90°-53.1° = 36.9°.[11] 120k/500 = 240. Answer is $240\angle 36.9^o$. Alternatively, for $\tan^{-1}\frac{R}{X}$, set 400 (C scale) over 300 (D scale), note D scale reading at index, move HL to value on C scale, note phase angle on reverse T scale as 36.9°.
Example: 2-branch circuit, Z1=50+j25, Z2=60+j20. Determine R and X separately. $R_t = \frac{50 \times 60}{110} = 27.3$, $X_t = \frac{25 \times 20}{45} = 11.1$. Z_T = 27.3+j11.1. Phase angle = $\tan^{-1}\theta = \frac{X}{R} = 22.13$.

Parallel circuits, total current method:[12] Using above example:

1) Find polar form of each branch: Z_1=50+j25 = $56\angle 26.6^o$, Z_2=60+j20 = $63.2\angle 18.4^o$.

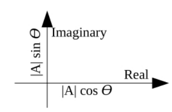
[11] 53.1 is found by $\tan^{-1}\frac{X}{R}$.
[12] Mathematics for Electricians and Radiomen, Nelson M. Cooke, First Edition, 1942, page 470.

2) Divide assumed or actual voltage by vector impedance. Angle becomes opposite in sign: $\frac{100}{56} = 1.79\angle - 26.6^o$, $\frac{100}{63.2} = 1.58\angle - 18.4^o$.
3) Convert currents to rectangular forms (see below) and add for total current:

$$1.79 - 26.6^o = 1.60 - j0.801 \quad (1)$$
$$1.58 - 18.4^o = 1.50 - j0.499 \quad (2)$$
$$I_{total} = 3.10 - j1.300 \quad (3)$$

4) Find polar form of I_{total}: 3.10-j1.30 = 3.36 -22.75°.
5) Divide voltage by vector current to determine joint vector impedance of the parallel combination. Angle becomes opposite in sign: $\frac{100}{3.36}$ = total impedance of $29.76\angle 22.75^o$.
6) Resolve the joint impedance into an equivalent series circuit if desired to add to other series impedances: $Z(\cos\theta + j\sin\theta)$.[13] Set index to 29.76 (D scale), slide HL to cos 22.75°, read 27.4 (D scale). Slide HL to sin 22.75°, read 11.51 (D scale). Series equivalent is 27.4+j11.51.

Polar to rectangular transformation: $Z(\cos\theta + j\sin\theta)$. Set index over Z on D scale, HL over cosine of angle (S scale, R to L). Read resistive Z on D scale. Move HL over sine of angle (S scale, L to R), read reactive Z on D scale. If angle negative, write as r-jx.



Example: Z of $50\angle 35^o$. Index on 50 (D scale), HL over cosine 35° (S scale, R to L), read 40.95. Move HL to sine 35° (S scale, L to R), read j28.65 (D Scale).
Example: Z of $12\angle - 42^o$. Index on 12 (D scale), HL over cosine 42° (S scale, R to L), read resistance 8.92 (D scale). HL to sin 42°, read negative reactance -j8.03. Answer 8.92-j8.03.

Table of Derivatives.

f(x)	f'(x)	f(x)	f'(x)
x	1	sin x	cos x
x^2	2x	cos x	-sin x
X^3	$3x^2$	e^x	e^x
x^n	nx^{n-1}	constant	0

Example: f(x) = x^3 - $6x^2$ + 9x + 1, derivative f'(x) = $3x^2$ - 12x + 9.
Example: derivative of $3x^2$ becomes 6x = 6(1) = 6.
Example: derivative of area, πr^2 is circumference, $2\pi r$, or πd.

[13] Standard Handbook for Electrical Engineers, 11th edition, 1978, page 2-37.

Dimensional Analysis and Conversions

Dimensional Unit Definitions.

Unit	Definition	Unit	Definition
M	mass	L	length
T	time	LT^{-1}	speed or velocity
L^2	area	L^3	volume
L/T or LT^{-1}	velocity	LT^{-2}	acceleration
MLT^{-2}	force	T^{-1}	frequency or angular velocity
I	electrical current	Q	charge rate or temperature
V	voltage	$VQ^{-1}T$	resistance
$V^{-1}Q$	capacitance	$VQ^{-1}T^2$	inductance
VQT^{-1}	power	ML^{-3}	density

Conversion Units.

Unit	Unit	Slide rule C over D	Dim. Unit
1 US gallon	3.7854 liters	37 over 140	L^3
1 US gallon	231 cu.in.	3 over 13	L^3
1 pound	0.4536 kg.	280 over 127	M
1 kg.	2.205 lb.	34 over 75	M
1 ton	907.2 kg.	7 over 6350	M
1 mi/hr.	1.467 ft./sec.	15 over 22	LT^{-1}
1 mi./hr.	88 ft./min.	1 over 88	LT^{-1}
1 mi./hr.	0.447 m./sec.	38 over 17	LT^{-1}
1 mi./hr.	26.84 m./min.	5 over 134	LT^{-1}
1 ft./min.	0.00508 m./sec.	6300 over 32	LT^{-1}
1 US gal/mile	2.3521 liter/km.	17 over 40	L^3/L ?
1 lb./in.^{2} (psi)	0.0703 kg./cm.^{2}	640 over 45	MLT^{-2}
1 lb./in.^{2} (psi)	2.036 inHg.	55 over 27	MLT^{-2}
1 lb./in.^{2} (psi)	2.307 ft. H^2O	26 over 60	MLT^{-2}
1 cu./ft. H^2O	62.43 lb.	17 over 1050	L^3
1 cu./ft. H^2O	28.32 kg.	12 over 340	L^3
1 gallon H^2O	8.345 lb.	3 over 25	M
1 H.P.	746 W	67 over 50	ML^2/T^3
1 H.P.	33,000 ft. lb./min.	1 over 33	ML^2/T^3
1 foot pound	0.1383 kg./m.	340 over 47	MLT^{-2}
1 foot	0.3048 meter	292 over 89	L
1000 mils (1 in.)	25.4 mm.	94 over 37	L
1 meter	39.4 in.	66 over 2600	L
1 meter	3.28 ft.	32 over 105	L
1 yard	0.9144 meter	35 over 32	L
1 mile	1.6093 km.	87 over 140	L
1 knot	1.151 mi./hr.	33over 38	LT^{-1}
1 sq. in.	6.4516 sq. cm.	31 over 200	L^2

Unit	Unit	Slide rule C over D	Dim. Unit
1 sq. ft.	0.0929 sq. m.	140 over 13	L^2
1 sq. yd.	0.8361 sq. m.	61 over 51	L^2
1 cu. in.	16.387 cu. cm.	36 over 590	L^3
1 cu. in.	0.00433 US gal.	6700 over 29	L^3
1 cu. ft.	0.0283 cu. m.	106 over 3	L^3
1 cu. ft.	7.481 US gal.	234 over 1750	L^3
1 cu. ft.	28.32 liters	3 over 85	L^3
Diagonal of circle	Circumference of circle	226 over 710	L
Side of square	Diagonal of square	70 over 99	L

SIGNED NUMBERS				
SAME SIGN	+	SAME SIGN	=	SAME SIGN
SAME SIGN	-	SAME SIGN	=	Lg - Sm, Use Sign of Larger
DIFFERENT	+	DIFFERENT	=	Lg - Sm, Use Sign of Larger
DIFFERENT	-	DIFFERENT	=	Add, Use Sign of 1st number.
SAME	* or ÷	SAME	=	Answer positive.
DIFFERENT	* or ÷	DIFFERENT	=	Answer negative.
FRACTIONS				
FRACTION	+ or -	FRACTION	=	FRACTION
(MUST HAVE SAME DENOMINATOR - “LCD”)				
FRACTION	*	FRACTION	=	FRACTION
(MAY HAVE DIFFERENT DENOMINATORS)				
FRACTION	÷	FRACTION	=	FLIP 2ND, THEN MULTIPLY
(MAY HAVE DIFFERENT DENOMINATORS)				

EXPONENTS	
$x^0 = 1(x \neq 0)$	$\frac{x^n}{x^n} = 1(x \neq 0)$
$x^m x^n = x^{m+n}$	$\frac{x^m}{x^n} = x^{m-n}$
$x^{-n} = \frac{1}{x^n}$	$(xy)^n = x^n y^n$
$(x^m)^n = x^{mn}$	$(\frac{x}{y})^n = \frac{x^n}{y^n}$

Specialty Slide Rules

N-515-T Electronic Slide Rule Special Scales.

Resonance Decimal Point Locator on back of rule provides approximate solutions, making it unnecessary to convert units. Then the front can give more precise solutions using the (Lr), (Cr), f_R and D scales. Set one value on upper body, the other on "**Resonance**" section of slide, read frequency on lower body by aligning HL with either f_{Hz}, f_{kHz}, or f_{MHz}. Arrows inline with each scale indicate direction of scale progression. To find a component value, set HL to frequency on lower body, align slide f_{kHz} arrow under HL, move HL to one component value on upper body, read other value on slide.

Example: Find resonance of 40mH with 0.03 uF. Set HL to 40 mH on upper body, slide 0.03 uF under HL, move HL to fkHz arrow and read ≈ 5 kHz. For precise answer, set 4 or 40 on front (L_r) scale and 3 or 30 on (C_r) scale. Two values are possible (14.5, 4.59); if one is not in range of previous answer use other half of the (C_r) slide. Answer: 4.594 kHz on D scale.

Example: Find capacitance for frequency of 8 MHz with 15 uH inductance. Set HL between 6 and 10 on lower body f scale. Align slide f_{MHz} arrow under HL, move HL to 15 uH on upper body, read ≈ 30 pF or ≈ .03 mF on slide. Find precise value by setting index to 8 (D scale), HL to 150 (or 15) on (L_r) scale, reading 26.4 pF on (C_r) scale.

Reactance Decimal Point Locator provides approximate X_L or X_C values using the "**Reactance**" section. Place slide frequency opposite given value of inductance or capacitance on upper body. Read reactance on lower body opposite appropriate X_L or X_C arrow on slide. Then the front gives precise solutions using the (f_X) and (L_X or C_X) scales and X_L or X_C arrows.

Example: Find X_C of .005 uF capacitor at 3 MHz. Set HL over 0.005 uF on upper body, move 3 MHz on slide under HL, read ≈ 10 Ω (X_C scale) on lower body at slide's X_C arrow. For precise answer, set HL to 3 on (f_X) scale, slide 5 of (L_X or C_X) scale under HL, read 10.61 Ω on C scale at D scale's X_C arrow.

Example: Find inductance value for X_L of 4 MΩ at 3 MHz. Set X_L, MΩ arrow at ≈ 4 on lower body X_L scale. HL to ≈ 3 MHz on slide, read ≈ 200 mH on upper body. For precise answer, HL to 3 on (f_X) scale, slide X_L arrow (index) to 4 (D scale), read 212 mH on slide's (L_X or C_X) scale.

H Scale. The LC product is L x C. Set HL to frequency on D scale, the product is shown on the H scale. Used primarily for resonance calculations.

Example: For L=10uH, C=10pF, $f_R = \frac{0.159}{\sqrt{LC}}$, where LC=100, set HL to 100 (H scale), read 15.9 (kHz) on D scale.
Example: Where 8uH and 10pf (10 x 8) = 80, setting HL to 80 of H scale, read 17.8 (kHz) on D scale.[14]

2π Scale. If HL is set over a certain value on the 2π scale, 2π times that value will appear under the HL on the D scale. Or, any number on the D scale will be divided by 2π and shown on the 2π scale. Used primarily for reactance calculations. 360° / 2π = 57.3° (2π radians = 360°)[15]

[14] See *Resonant Frequency* section under **Electronics** for additional decimal placement information.
[15] See also "Radian Measure" section.

Examples: 1 on 2π scale = 6.28 on D scale. 4π= 12.566. $2\pi^2$ = 19.739 (HL set on π). $\frac{1}{2\pi}$ = 0.159.[16]

[16] Reciprocal of 6.28. See also "Capacitive Reactance" section.