A Designer's Guide to Microstrip Design

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This effort is an attempt to recreate a portion of the above titled article[1], (Bahl and Trivedi 1977), from the May 1977 issue of Microwaves Journal. I originally came across the reference in (Pozar 2012) Chapter 3, where I was investigating frequency effects on microstrip lines, and wanted to repeat some of the explanation and formulae here.

This is not an announcement of a revolutionary *new idea* for microstrip circuit design. It is, however, a compendium of the *best ideas* presented over the past several years on design considerations for this ubiquitous transmission medium.

Microstrip technology is quite mature, offering a superior blend of performance characteristics to the designer of microwave integrated circuits. So today, the problem is not that the circuit designer lacks information concerning this transmission medium, but that too much information is available, scattered throughout too many journals.

Since field lines between the strip and the ground plane are not contained entirely in the substrate, the propagating mode along the strip is not purely transverse electromagnetic (TEM) but quasi-TEM. Assuming the quasi-TEM mode of propagation, the phase velocity in microstrip is given by

$$V_p = \frac{c}{\sqrt{\varepsilon_{eff}}} \tag{1}$$

where c is the velocity of light[2], and ε_{eff} is the effective dielectric constant of the substrate material. The effective dielectric constant is lower than the relative dielectric constant, ε_r, of the substrate, and takes into account external fields.

The wavelength, λ_g, in microstrip line is given by

$$\lambda_g = \frac{V_p}{f}\ (in\, m/s) \tag{2}$$

where V_p is given by Eq. 1 and f is frequency.

The characteristic impedance of the transmission line is given by

$$Z_o = \frac{1}{V_p C} \tag{3}$$

where C is the capacitance per unit length of the line.

The analysis for the evaluation of ε_{eff} and C based on quasi-TEM mode is fairly accurate at lower microwave frequencies. At higher frequencies, the ratio of longitudinal-to-traverse electric field components becomes significant and the propagating mode can no longer be considered quasi-TEM. Analysis of this "hybrid mode" is far more rigorous.

Closed-form expressions. Closed form expressions by Hammerstad[3] for Z_o and ε_{eff} include useful relationships defining both characteristic impedance and effective dielectric constant:
For W/h $\leq$ 1,

$$Z_o = \frac{60}{\sqrt{\varepsilon_{eff}}} ln(8h/W + 0.25W/h) \tag{4}$$

where:

$$\varepsilon_{eff} = \frac{e_r+1}{2} + \frac{e_r-1}{2}[(1+12h/W)^{-1/2} + 0.04(1-W/h)^2] \tag{5}$$

For W/h $\geq$ 1,

$$Z_o = \frac{120\pi/\sqrt{\varepsilon_{eff}}}{W/h + 1.393 + 0.667 ln(W/h + 1.444)} \tag{6}$$

where:

$$\varepsilon_{eff} = \frac{e_r+1}{2} + \frac{e_r-1}{2}(1+12h/W)^{-1/2} \tag{7}$$

Hammerstad notes that the maximum relative error in ε_{eff} and Z_o is less than $\pm$ 0.5 percent and 0.8 percent, respectively, for 0.05 $\leq$ W/h $\leq$ 20 and $\varepsilon_r \leq 16$. His expressions for W/h in terms of Z_o and ε_r are:
For W/h $\leq$ 2,

$$W/h = \frac{8e^A}{e^{2A} - 2} \tag{8}$$

[1]Submitted by Dr. I. J. Bahl, and D. K. Trivedi, Research Engineers, Indian Institute of Technology, Advanced Centre For Electronic Systems, department of Electrical Engineering, Kanpur-208016, India.

[2]Lightspeed is 2.99792458e8 m/s.

[3]E.O Hammerstad, "Equations For Microstrip Circuit Design," Proc. European Microwave Conference, Hamburg (Germany), pp. 268-272, (September 1975).

For W/h ≥ 2,

$$W/h = \frac{2}{\pi}[B - 1 - ln(2B-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left(ln(B-1) + 0.39 - \frac{0.61}{\varepsilon_r}\right)] \quad (9)$$

where:

$$A = \frac{Z_o}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}(0.23 + 0.11/\varepsilon_r)$$
$$B = \frac{377\pi}{2Z_o\sqrt{\varepsilon_r}}$$

These expressions provide the same accuracy as Eqs. 4, 5, 6 and 7.

The results discussed above assume a two-dimensional strip conductor. But in practice, the strip is three-dimensional; its thickness must be considered. However, when t/h ≤ 0.005, 2 ≤ ε_r ≤ 10, and 0.1 ≤ W/h ≤ 5, the agreement between experimental and theoretical (t/h=0) results is excellent.

The zero-thickness (t/h=0) formulas given above can also be modified to consider the thickness of the strip when the strip width, W, is replaced by an effective strip width, W_e. Expressions for W_e are:
For W/h ≥ 1/2π,

$$\frac{W_e}{h} = \frac{W}{h} + \frac{t}{\pi h}\left(1 + ln\frac{2h}{t}\right) \quad (10)$$

For W/h ≤ 1/2π,

$$\frac{W_e}{h} = \frac{W}{h} + \frac{t}{\pi h}\left(1 + ln\frac{4\pi W}{t}\right) \quad (11)$$

Additional restrictions for applying Eqs 10 and 11 are t ≤ h and t < W/2. Typical strip thickenss varies from 0.0002 to 0.0005 inch (5.1 um to 12.7 um) for metalized alumina substrate, and from 0.001 to 0.003 inch (25 um to 76 um)[4] for low-dielectric substrates.

Microwave enclosures tend to lower impedance and effective dielectric constant. But when the ratio of the distance between the lower and upper walls to substrate thickness is larger than five, and the sidewall spacing is five times the strip width, the enclosure effect is negligible on microstrip characteristics.

Higher frequency dispersion. The formulas for characteristic impedance and effective dielectric constant presented thus far have been based on a quasi-TEM mode of propagation. At lower frequencies, this is a good static approximation of a dynamic structure. However, at higher frequencies, the effective dielectric constant and characteristic impedance of a microstrip line begins to change as frequency increases, making the transmission line dispersive. This dispersive characteristic is due to propagation of hybrid modes.

The frequency dependence of the effective dielectric constant describes the influence of dispersion on the phase velocity, whereas the frequency dependence of the effective width describes the influence of the dispersion on the characteristic impedance. The phase velocity in microstrip line decreases with increasing frequency, hence ε_{eff} increases with frequency. The characteristic impedance of microstrip line increases with frequency, so the effective width must decrease with frequency.

Fortunately, changes in ε_{eff} and Z_o with frequency are very small. However, the frequency below which dispersion effects may be neglected is given by the relation

$$f_o(GHz) = 0.3\sqrt{\frac{Z_o}{h\sqrt{\varepsilon_r - 1}}}\,(h\;in\;cm) \quad (12)$$

Equation 12 shows that f_o is higher for high-impedance lines on thin substrates.

Analytical formulas for dispersion which agree closely with both experimental and numerical results have appeared just recently. The analytical expressions by Getsinger[5] for the dispersion in ε_{eff} is given by

$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}}{1 + G(f/f_p)^2} \quad (13)$$

where:

$$f_p = \frac{Z_o}{8\pi h} \qquad G = 0.6 + 0.009Z_o$$

Here, frequency, f, is in GHz and substrate thickness, h, in cm. It can be seen from Eq. 13 that for $f_p \gg f$, $\varepsilon_{eff}(f) = \varepsilon_{eff}$. In other words, high-impedance lines on thin substrates are less dispersive. There is a close agreement between the calculated values and experimental values.

Closed form dispersive expressions for $Z_o(f)$ based on a parallel-plate model of microstrip line have been reported[6]
These expressions are:

$$Z_o(f) = \frac{377h}{W_{eff}(f)\sqrt{\varepsilon_{eff}}(f)} \quad (14)$$

[4] 35 um is average thickness for 1 oz copper traces.

[5] W.J. Getsinger, "Microstrip Dispersion," Proc. IEEE, Vol. 60, pp. 144-146, (January, 1972).

[6] R.P. Owens, "Predicted Frequency Dependence Of Microstrip Characteristic Impedance Using The Planar-Waveguide Model," Electron. Letter., Vol. 12, pp 269-270, (May 27, 1976).

The effective width, $W_{eff}(f)$, is given by

$$W_{eff}(f) = W + \frac{W_{eff}(0) - W}{1 + (f/f_p)^2} \quad (15)$$

where $W_{eff}(0)$ is obtained from Eq. 14 when f = 0.

The increase in $Z_o(f)$ (for $\varepsilon_r = 10$, and W/h = 1) is only 4 percent from DC to 10 GHz, which is quite small. This change connot be confirmed experimentally since, at 10 GHz, transitions pose a considerable problem in accurate measurement. Therefore, the effect of dspersion on Z_o can be generally neglected.

Attenuation losses. Attenuation constant, α, is one of the most important characteristics of any transmission line. There are two sources of dissipative losses in a microstrip circuit: conductor loss and substrate dielectric loss.

Assuming a uniform current distribution across strip width and ground plane, conductor loss may be approximated as:

$$\alpha_c = \frac{8.68}{Z_o W} R_s \, dB/cm \quad (16)$$

The surface resistivity, R_s, for the conductor is given by $R_s = \sqrt{\pi f \mu_o / \sigma}$ where μ_o is the free space permeability, 12.5663e-7, and σ is the conductivity of the microstrip material[7]. This expression is only practical if non-uniform current distribution is considered.

Expressions for the conductor loss derived by Pucel[8] are very accurate.
For $W/h \leq 1/2\pi$,

$$\alpha_c = \frac{8.68 R_s}{2\pi Z_o h} * P * \left[1 + \frac{h}{W_e} + \frac{h}{\pi W_e}\left(ln\frac{4\pi W}{t} + \frac{t}{W}\right)\right] \quad (17)$$

For $1/2\,\pi < W/h \leq 2$,

$$\alpha_c = \frac{8.86 R_s}{2\pi Z_o h} * P * Q \quad (18)$$

For $W/h \geq 2$,

$$\alpha_c = \frac{8.68 R_s}{Z_o h} * Q * [\frac{W_e}{h} + \frac{2}{\pi}\, ln(2\pi\, e\left(\frac{W_e}{2h}\right) + 0.94)]^{-2} \left[\frac{W_e}{h} + \frac{W_e/\pi h}{\frac{W_e}{2h} + 0.94}\right] \quad (19)$$

where:

$$P = 1 - \left(\frac{W_e}{4h}\right)^2 \qquad Q = 1 + \frac{h}{W_e} + \frac{h}{\pi W_e}\left(ln\frac{2h}{t} - \frac{t}{h}\right)$$

For a fixed characteristic impedance, conductor loss decreases inversely with substrate thickness and increases with the square root of frequency.

The dielectric loss in microstrip line is an important parameter when microwave circuits requiring small attenuation are considered. Welch and Pratt[9] and Schneider[10] derived the expressions for the attenuation constant for a dielectric with loss tangent, $tan\delta$, given below:

$$\alpha_d = 27.3 \frac{\varepsilon_r}{(\varepsilon_{eff})^{1/2}} * \frac{\varepsilon_{eff} - 1}{\varepsilon_r - 1} * \frac{tan\delta}{\lambda_o} \, dB/cm \quad (20)$$

where λ_o is the free space wavelength.
For $\sigma \neq 0$,

$$\alpha_d = 4.34 \frac{\varepsilon_{eff} - 1}{\sqrt{\varepsilon_{eff}}(\varepsilon_r - 1)} \left(\frac{\mu_o}{\varepsilon_o}\right)^{1/2} \sigma \, dB/cm \quad (21)$$

where ε_o is the free space permittivity, 8.854187817e-12[11].

Dielectric losses are normally very small compared with conductor losses for dielectric substrate. Dielectric losses in silicon substrates, however, are usually in the same order as, or larger than conductor losses. The reason for this is that resistivities higher than few hundred ohm-cm are difficult to maintain for Si. However, higher resistivity can be maintained in GaAs, and hence the losses are less for this material.

Quality factor depends on substrate. The quality factor, Q, of a microstrip line can be related to the total losses in the line by

$$Q_T = \frac{\beta}{2\alpha_T} \quad (22)$$

where Q_T is the total resonator Q, α_T is the total loss in the resonator and $\beta = 2\pi/\lambda_g$. Microstrip line Qs are lower than the Qs of coaxial or waveguide transmission lines.

[7] The conductivity of copper is 5.959475566e7 Ω/m at room temperature (293° K). Handbook of Chemistry and Physics, 88th Ed. p. 12-39 (2007-2008)

[8] R.A. Pucel, D.J. Masse and C.P. Hartwing, "Losses in Microstrip," IEEE Trans. Microwave Theory Tech., Vol. MTT-16, pp. 342-350, (June, 1968).

[9] J.D. Welch and H.J. Pratt, "Losses In Microstip Trnsmission Systems For Integrated Microwave Circuits," NEREM Rec. Vol. 8, pp. 100-101, (1966).

[10] M.V. Schneider, "Dielectric Loss In Integrated Microwave Circuits," Bell Syst. Tech. J. 48, No. 7, pp. 2325-2332, (September, 1969).

[11] Free space permittivity, also known as electric constant, $1/\mu_o c^2$. $\mu_o c$ is the characteristic impedance of vacuum, Z_o = 376.730313461 Ω.

When the losses in a resonant line are considered (such as $\lambda_g/2$ or $\lambda_g/4$ resonators) another loss factor, α_r, due to radiation at the discontinuities must also be considered. The corresponding radiation Q-factor is given by,

$$Q_r = \frac{Z_o}{480\pi(h/\lambda_o)^2 F} \tag{23}$$

where:

$$F = \frac{\varepsilon_{eff}(f)+1}{\varepsilon_{eff}(f)} - \frac{(\varepsilon_{eff}(f)-1)^2}{2(\varepsilon_{eff}(f)^{2/3}} * ln\frac{\sqrt{\varepsilon_{eff}(f)}+1}{\sqrt{\varepsilon_{eff}(f)}-1}$$

Note that the effect of dispersion is considered, as described by Eq. 13. The total Q of the resonator can be expressed by,

$$\frac{1}{Q_T} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r} \tag{24}$$

Here, Q_c, Q_d and Q_r are the quality factors corresponding to conductor, dielectric and radiation losses, respectively. Finally, the circuit quality factor, Q_o, can be defined as,

$$\frac{1}{Q_o} = \frac{1}{Q_c} + \frac{1}{Q_d} = \frac{\lambda_o(\alpha_c+\alpha_d)}{\pi\sqrt{\varepsilon_{eff}}} \tag{25}$$

A quarter-wave, 50-ohm resonator on 25-mil-thick (0.635 mm) alumina substrate has a Q_o of about 240 at 2 GHz and 550 at 10 GHz, whereas Q_T is 230 at 2 GHz and nearly 160 at 10 GHz. This is due to the fact that radiation losses are higher than conductor and dielectric losses at higher frequencies. A quarter-wave, 50-ohm resonator on 10-mil (0.254 mm) GaAs substrate has Q_o of about 82 at 2 GHz and 160 at 10 GHz, whereas Q_T is 82 at 2 GHz and nearly 145 at 10 GHz. This is due to the fact that radiation losses are smaller than conductor and dielectric losses for thin substrates at higher frequencies.Thus, the commonly accepted rule for high-Q microstrip circuits using thick substrates does not apply due to high radiation losses incurred under this condition.

Moding limits high-frequency operation. Maximum frequency of operation in microstrip line is limited by the excitation of spurious modes in the form of surface waves and transverse resonances. Surface waves are TM and TE modes which propagate across a dielectric substrate with ground plane. The frequency at which significant coupling occurs between the quasi-TEM mode and the lowest-order surface wave mode is given by,

$$f_T = \frac{c}{2\pi h}\sqrt{\frac{2}{\varepsilon_r - 1}} * \tan^{-1}(\varepsilon_r) \tag{26}$$

For $\varepsilon_r > 10$, Eq. 26 reduces to,

$$f_T(GHz) = \frac{10.6}{h\sqrt{\varepsilon_r}}\,(h\,is\,in\,cm) \tag{27}$$

Cutoff frequency, f_T, decreases when either the substrate thickness or dielectric constant is increased.

Thus three limitations–maximum substrate thickness, minimum Q and surface wave excitation–define a region of useful microstrip line operation. From the usable region, one obtains a range of substrate thickness which should be used for microstrip line circuits.

For $\varepsilon_r = 9.7$, this range is:

- $0.23 \le$ h ≤ 1.8 cm @ 2 GHz
- $0.01 \le$ h ≤ 0.36 cm @ 10 GHz
- $0.01 \le$ h ≤ 0.17 cm @ 20 GHz

In addition to the conductor and dielectric losses (Q_o), the maximum Q of microstrip is also limited by radiation losses from discontinuities.

NOTE: I have not attempted to reproduce the various charts that are part of the article, nor the fabrication tolerances and power handling sections. Some sections were reduced in content and some metric (SI) conversions were added to SAE values. I have duplicated the two-column layout for aesthetic purposes.This document format and formulae were produced using (R Core Team 2021) and (Allaire et al. 2021).

Allaire, JJ, Yihui Xie, Jonathan McPherson, Javier Luraschi, Kevin Ushey, Aron Atkins, Hadley Wickham, Joe Cheng, Winston Chang, and Richard Iannone. 2021. *Rmarkdown: Dynamic Documents for r*. https://CRAN.R-project.org/package=rmarkdown.

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